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VOLUME 2

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V. James Abraham

R. Lakshmi

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MATHEMATICS

STANDARD SEVEN

TERM I

Volume 2

Textbook Team

CHAIRPERSON

N. Varadarajan
Associate Professor in Mathematics
Presidency College (Autonomous)
Chennai - 600 005.

REVIEWERS

N. Ramesh
Associate Professor in Mathematics
Govt. Arts College (Autonomous)
Nandanam, Chennai - 600 035.

K. Govindan
Headmaster, Govt. Girls Hr. Sec. School
Paradharami - Gudiyatham
Vellore District.

AUTHORS

A. Yuvaraj
Supervisor
Block Resource Centre, Mathur
Krishnagiri District.

G. Palani
Post Graduate Teacher
Govt. Hr. Sec. School (ADW)
Nagalkeni, Chennai - 600 044.

M. Shubhashiny
Graduate Teacher
Govt. Hr. Sec. School
Somangalam, Kancheepuram Dist.

R. Karpagavalli
Post Graduate Teacher
Padma Sarangapani Matric. Hr. Sec. School
Virugambakkam, Chennai - 600 092.

B. Malarvizhi
Graduate Teacher
SBOA Matric. Hr. Sec. School
Chennai - 600 101.

D. Ulaganathan
Graduate Teacher
Govt. Girls Hr. Sec. School
Paradharami - Gudiyatham, Vellore District.

Laser Typeset & Layout : **Shiv Anand, V.James Abraham, R. Lakshmi,** Perambur.

Chapter 1

1 REAL NUMBER SYSTEM

No World without Water

No Mathematics without Numbers

1.1 Introduction

In the development of science, we should know about the properties and operations on numbers which are very important in our daily life. In the earlier classes we have studied about the whole numbers and the fundamental operations on them. Now, we extend our study to the integers, rationals, decimals, fractions and powers in this chapter.

Numbers

In real life, we use Hindu Arabic numerals - a system which consists of the symbols 0 to 9. This system of reading and writing numerals is called, “Base ten system” or “Decimal number system”.

1.2 Revision

In VI standard, we have studied about Natural numbers, Whole numbers, Fractions and Decimals. We also studied two fundamental operations addition and subtraction on them. We shall revise them here.

Natural Numbers

Counting numbers are called natural numbers. These numbers start with smallest number 1 and go without end. The set of all natural numbers is denoted by the symbol ‘N’.

$N = \{1, 2, 3, 4, 5, ...\}$ is the set of all natural numbers.

Whole numbers

Natural numbers together with zero (0) are called whole numbers. These numbers start with smallest number 0 and go without end. The set of all whole numbers is denoted by the symbol ‘W’.

$W = \{0, 1, 2, 3, 4, 5, ...\}$ is the set of all whole numbers.

Integers

The whole numbers and negative numbers together are called integers. The set of all integers is denoted by Z.

$Z = \{... -2, -1, 0, 1, 2, ...\}$ is the set of all integers (or) $Z = \{0, \pm 1, \pm 2, ...\}$ is the set of all Integers.

Do you know?

Ramanujan, the greatest Mathematician was born at Erode in Tamil Nadu.

1.3 Four Fundamental Operations on Integers

(i) Addition of Integers

Sum of two integers is again an integer.

For example,

i) $10 + (-4) = 10 - 4 = 6$

ii) $8 + 4 = 12$

iii) $6 + 0 = 6$

iv) $6 + 5 = 11$

v) $4 + 0 = 4$

(ii) Subtraction of integers

To subtract an integer from another integer, add the additive inverse of the second number to the first number.

For example,

i) $5 - 3 = 5 +$ (addditive inverse of 3) $= 5 + (-3) = 2$.

ii) $6 - (-2) = 6 +$ (addditive inverse of (-2)) $= 6 + 2 = 8$.

iii) $(-8) - (5) = (-8) + (-5) = -13$.

iv) $(-20) - (-6) = -20 + 6 = -14$.

(iii) Multiplication of integers

In the previous class, we have learnt that multiplication is repeated addition in the set of whole numbers. Let us learn about it now in the set of integers.

Rules :

1. The product of two positive integers is a positive integer.
2. The product of two negative integers is a positive integer.
3. The product of a positive integer and a negative integer is a negative integer.

Example

i) $5 \times 8 = 40$

ii) $(-5) \times (-9) = 45$

iii) $(-15) \times 3 = -(15 \times 3) = -45$

iv) $12 \times (-4) = -(12 \times 4) = -48$

Try these

1) $0 \times (-10) =$
2) $9 \times (-7) =$
3) $-5 \times (-10) =$
4) $-11 \times 6 =$

Activity

Draw a straight line on the ground. Mark the middle point of the line as ‘0’ (Zero). Stand on the zero. Now jump one step to the right on the line. Mark it as + 1. From there jump one more step in the same direction and mark it as + 2. Continue jumping one step at a time and mark each step (as + 3, + 4, + 5, ...). Now come back to zero position on the line. Move one step to the left of ‘0’ and mark it as – 1. Continue jumping one step at a time in the same direction and mark the steps as – 2, – 3, – 4, and so on. The number line is ready. Play the game of numbers as indicated below.

i) Stand on the zero of the number line facing right side of 0. Jumping two steps at a time. If you continue jumping like this 3 times, how far are you from ‘0’ on number line?

ii) Stand on the zero of number line facing left side of 0. Jump 3 steps at a time. If you continue jumping like this 3 times, how far are you from ‘0’ on the number line?

Activity

×	4	– 6	– 3	2	7	8
– 6	– 24					
– 5			15			– 40
3					21	

Example 1.1

Multiply (– 11) and (– 10).

Solution

$-11 \times (-10) = (11 \times 10) = 110$

Example 1.2

Multiply (– 14) and 9.

Solution

$(-14) \times 9 = -(14 \times 9) = -126$

Example 1.3

Find the value of 15 × 18.

Solution

15 × 18 = 270

Example 1.4

The cost of a television set is ₹5200. Find the cost of 25 television sets.

Solution

The cost one television set = ₹5200

∴ The cost of 25 television set = 5200 × 25

= ₹130000

Activity

Multiplication of integers through number patterns

Multiplying a negative integer by another negative integer :

Eg. To explain (-2) × (-2) = 4 through number pattern.

Activity :

(+2) × (+1) = 2 (Reduce the multiplicand each time by one)

(+1) × (+1) = 1

(0) × (+1) = 0

(–1) × (+1) = –1

(–2) × (+1) = –2

Reduce the multiplier each time by one

(–2) × (0) = 0

(–2) × (–1) = 2

(–2) × (–2) = 4

Exercise 1.1

1. Choose the best answer:

i) The value of multiplying zero with any other integer is a

(A) positive integer (B) negative integer (C) 1 (D) 0

ii) – 15^2 is equal to

(A) 225 (B) – 225 (C) 325 (D) 425

iii) – 15 × (– 9) × 0 is equal to

(A) – 15 (B) – 9 (C) 0 (D) 7

iv) The product of any two negative integers is a

(A) negative integer (B) positive integer

(C) natural number (D) whole number

2. Fill in the blanks:

i) The product of a negative integer and zero is __________.

ii) __________ $\times(-14) = 70$

iii) $(-72)\times$ __________ $= -360$

iv) $0\times(-17) =$ __________.

3. Evaluate:

i) $3\times(-2)$ ii) $(-1)\times 25$ iii) $(-21)\times(-31)$

iv) $(-316)\times 1$ v) (– 16) × 0 × (– 18) vi) $(-12)\times(-11)\times 10$

vii) $(-5)\times(-5)$ viii) 5×5 ix) $(-3)\times(-7)\times(-2)\times(-1)$

x) $(-1)\times(-2)\times(-3)\times 4$ xi) $7\times(-5)\times(9)\times(-6)$

xii) $7\times 9\times 6\times(-5)$ xiii) 10 × 16 × (– 9)

xiv) 16 × (– 8) × (– 2) xv) (– 20) × (– 12) × 25

xvi) 9 × 6 × (– 10) × (– 20)

4. Multiply

i) (-9) and 15

ii) (-4) and (-4)

iii) 13 and 14

iv) (-25) with 32

v) (-1) with (-1)

vi) (-100) with 0

5. The cost of one pen is ₹15. What is the cost of 43 pens?

6. A question paper contains 20 questions and each question carries 5 marks. If a student answered 15 questions correctly, find his mark?

7. Revathi earns ₹ 150 every day. How much money will she have in 10 days?

8. The cost of one apple is ₹20. Find the cost of 12 apples?

(iv) Division of integers

We know that division is the inverse operation of multiplication.

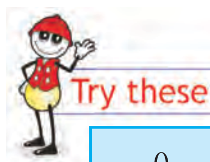
We can state the rules of division as follows:

$$\frac{\text{Positive integer}}{\text{Positive integer}} = \text{Positive number}$$

$$\frac{\text{Negative integer}}{\text{Negative integer}} = \text{Positive number}$$

$$\frac{\text{Positive integer}}{\text{Negative integer}} = \text{Negative number}$$

$$\frac{\text{Negative integer}}{\text{Positive integer}} = \text{Negative number}$$



a) $\frac{0}{10} =$ b) $\frac{9}{-3} =$

c) $\frac{-3}{-3} =$ d) $\frac{-10}{2} =$

Division by zero

Division of any number by zero (except 0) is meaningless because division by zero is not defined.

Example 1.5

Divide 250 by 50.

Solution

Divide 250 by 50 is $\frac{250}{50} = 5$.

Example 1.6

Divide (– 144) by 12.

Solution

Divide (– 144) by 12 is $\frac{-144}{12} = -12$.

Example 1.7

Find the value $\frac{15\times(-30)\times(-60)}{2\times10}$.

Solution

$$\frac{15\times(-30)\times(-60)}{2\times10} = \frac{27000}{20} = 1350.$$

Example 1.8

A bus covers 200 km in 5 hours. What is the distance covered in 1 hour?

Solution

Distance covered in 5 hours = 200 km.

$\therefore$ Distance covered in 1 hour = $\frac{200}{5}$ = 40 km.

Exercise 1.2

1. Choose the best answer:

i) Division of integers is inverse operation of

(A) addition (B) subtraction (C) multiplication (D) division

ii) 369 ÷ .... = 369.

(A) 1 (B) 2 (C) 369 (D) 769

iii) – 206 ÷ .... = 1.

(A) 1 (B) 206 (C) – 206 (D) 7

iv) – 75 ÷ .... = – 1.

(A) 75 (B) – 1 (C) – 75 (D) 10

2. Evaluate

i) $(-30)\div 6$ ii) $50 \div 5$ iii) $(-36)\div(-9)$

iv) $(-49)\div 49$ v) $12 \div[(-3)+1]$ vi) $[(-36)\div 6]-3$

vii) $[(-6)+7]\div[(-3)+2]$ viii) $[(-7)+(-19)]\div[(-10)+(-3)]$

ix) $[7+13]\div[2+8]$ x) $[7 + 23] \div [2 + 3]$

3. Evaluate

i) $\frac{(-1)\times(-5)\times(-4)\times(-6)}{2\times3}$ ii) $\frac{8\times5\times4\times3\times10}{4\times5\times6\times2}$ iii) $\frac{40\times(-20)\times(-12)}{4\times(-6)}$

4. The product of two numbers is 105. One of the number is (– 21). What is the other number?

1.3 Properties of Addition of integers

(i) Closure Property

Observe the following examples:

1. $19 + 23 = 42$
2. $-10 + 4 = -6$
3. $18 + (-47) = -29$

In general, for any two integers a and b, $a + b$ is an integer.

Therefore the set of integers is closed under addition.

(ii) Commutative Property

Two integers can be added in any order. In other words, addition is commutative for integers.

We have $8 + (-3) = 5$ and $(-3) + 8 = 5$

So, $8 + (-3) = (-3) + 8$

In general, for any two integers a and b we can say, $a + b = b + a$

Therefore addition of integers is commutative.

Are the following equal?

i) $(5) + (-12)$ and $(-12) + (5)$

ii) $(-20) + 72$ and $72 + (-20)$

(iii) Associative Property

Are the following pairs of expressions equal?

i) $7 + (5 + 4), (7 + 5) + 4$

ii) $(-5) + [(-2) + (-4)]$, $[(-5) + (-2)] + (-4)$

Observe the following example:

Consider the integers 5, – 4 and 7.

Look at $5 + [(-4) + 7] = 5 + 3 = 8$ and

$[5 + (-4)] + 7 = 1 + 7 = 8$

Therefore, $5 + [(-4) + 7] = [5 + (-4)] + 7$

In general, for any integers a, b and c, we can say, $a + (b + c) = (a + b) + c$.

Therefore addition of integers is associative.

(iv) Additive identity

When we add zero to any integer, we get the same integer.

Observe the example: $5 + 0 = 5$.

In general, for any integer a, $a + 0 = a$.

Therefore, zero is the additive identity for integers.

i) $17 +$ ___ $= 17$

ii) $0 +$ ___ $= 20$

iii) $-53 +$ ___ $= -53$

Properties of subtraction of integers.

(i) Closure Property

Observe the following examples:

i) $5 - 12 = -7$

ii) $(-18) - (-13) = -5$

From the above examples it is clear that subtraction of any two integers is again an integer. In general, for any two integers a and b, $a - b$ is an integer.

Therefore, the set of integers is closed under subtraction.

(ii) Commutative Property

Consider the integers 7 and 4. We see that

$7 - 4 = 3$

$4 - 7 = -3$

$\therefore\ 7 - 4 \neq 4 - 7$

In general, for any two integers a and b

$a - b \neq b - a$

Therefore, we conclude that subtraction is not commutative for integers.

(iii) Associative Property

Consider the integers 7, 4 and 2

$7 - (4 - 2) = 7 - 2 = 5$

$(7 - 4) - 2 = 3 - 2 = 1$

$\therefore\ 7 - (4 - 2) \neq (7 - 4) - 2$

In general, for any three integers a , b and c

$a - (b - c) \neq (a - b) - c.$

Therefore, subtraction of integers is not associative.

Properties of multiplication of integers

(i) Closure property

Observe the following:

$$-10 \times (-5) = 50$$

$$40 \times (-15) = -600$$

In general, $a \times b$ is an integer, for all integers a and b.

Therefore, integers are closed under multiplication.

Are the following pairs equal?

i) $5 \times (-7)$, $(-7) \times 5$

ii) $9 \times (-10)$, $(-10) \times 9$

(ii) Commutative property

Observe the following:

$$5 \times (-6) = -30 \quad \text{and} \quad (-6) \times 5 = -30$$

$$5 \times (-6) = (-6) \times 5$$

Therefore, multiplication is commutative for integers.

In general, for any two integers a and b, $a \times b = b \times a$.

(iii) Multiplication by Zero

The product of any nonzero integer with zero is zero.

Observe the following:

$$5 \times 0 = 0$$

$$-8 \times 0 = 0$$

In general, for any nonzero integer a

$$a \times 0 = 0 \times a = 0$$

i) $0 \times 0 =$ ______

ii) $-100 \times 0 =$ ______

iii) $0 \times x =$ ______

(iv) Multiplicative identity

Observe the following:

$$5 \times 1 = 5$$

$$1 \times (-7) = -7$$

This shows that '1' is the multiplicative identity for integers.

In general, for any integer a we have

$$a \times 1 = 1 \times a = a$$

i) $(-10) \times 1 =$ ___

ii) $(-7) \times$ ___ $= -7$

iii) ___ $\times 9 = 9$

(v) Associative property for Multiplication

Consider the integers 2, – 5, 6.

Look at

$[2\times(-5)]\times 6 = -10\times 6$

$= -60$ and

$2\times[(-5)\times 6] = 2\times(-30)$

$= -60$

Thus $[2\times(-5)]\times 6 = 2\times[(-5)\times 6]$

So we can say that integers are associative under multiplication.

In general, for any integers a, b, c, $(a \times b) \times c = a \times (b \times c)$.

(vi) Distributive property

Are the following equal?

1. $4\times(5+6)$ and $(4\times 5)+(4\times 6)$
2. $3\times(7-8)$ and $(3\times 7)+(3\times -8)$
3. $4\times(-5)$ and $(-5)\times 4$

Consider the integers 12, 9, 7.

Look at

$12\times(9+7) = 12\times 16 = 192$

$(12\times 9)+(12\times 7) = 108+84 = 192$

Thus $12\times(9+7) = (12\times 9)+(12\times 7)$

In general, for any integers a, b, c.

$$a\times(b+c) = (a\times b)+(a\times c).$$

Therefore, integers are distributive under multiplication.

Properties of division of integers

(i) Closure property

Observe the following examples:

(i) $15 \div 5 = 3$

(ii) $(-3)\div 9 = \frac{-3}{9} = \frac{-1}{3}$

(iii) $7\div 4 = \frac{7}{4}$

From the above examples we observe that ***integers are not closed under division.***

(ii) Commutative Property

Observe the following example:

$8 \div 4 = 2$ and

$4 \div 8 = \frac{1}{2}$

$\therefore 8 \div 4 \neq 4 \div 8$

We observe that ***integers are not commutative under division.***

(iii) Associative Property

Observe the following example:

$12 \div (6 \div 2) = 12 \div 3 = 4$

$(12 \div 6) \div 2 = 2 \div 2 = 1$

$\therefore 12 \div (6 \div 2) \neq (12 \div 6) \div 2$

From the above example we observe that ***integers are not associative under division.***

Activity Divide the class into groups each group has to complete the given table using their own examples and then write true (or) false.

Properties of Integers	Closure Property	Commutative property	Associative property
Addition			
Subtraction			
Multiplication			
Division			

1.4 Fractions

Introduction

In the earlier classes we have learnt about fractions which included proper, improper and mixed fractions as well as their addition and subtraction. Now let us see multiplication and division of fractions.

Recall :

Proper fraction: A fraction is called a proper fraction if its Denominator > Numerator.

Example: $\frac{3}{4}, \frac{1}{2}, \frac{9}{10}, \frac{5}{6}$

Improper fraction: A fraction is called an improper fraction if its Numerator > Denominator.

Example : $\frac{5}{4}, \frac{6}{5}, \frac{41}{30}, \frac{51}{25}$

Mixed fraction : A fraction consisting of a natural number and a proper fraction is called a mixed fractions.

Example: $2\frac{3}{4}, 1\frac{4}{5}, 5\frac{1}{7}$

Think it : Mixed fraction = Natural number + Proper fraction

Discuss : How many numbers are there from 0 to 1.

Do you know?

All whole numbers are fractional numbers with 1 as the denominator.

Recall : Addition and subtraction of fractions.

Example (i)

Simplify: $\frac{2}{5}+\frac{3}{5}$

Solution

$$\frac{2}{5}+\frac{3}{5}=\frac{2+3}{5}=\frac{5}{5}=1$$

Example (ii)

Simplify: $\frac{2}{3}+\frac{5}{12}+\frac{7}{24}$

Solution

$$\frac{2}{3}+\frac{5}{12}+\frac{7}{24}=\frac{2\times 8+5\times 2+7\times 1}{24}$$
$$=\frac{16+10+7}{24}$$
$$=\frac{33}{24}=1\frac{3}{8}$$

Example (iii)

Simplify: $5\frac{1}{4}+4\frac{3}{4}+7\frac{5}{8}$

Solution

$$5\frac{1}{4}+4\frac{3}{4}+7\frac{5}{8}=\frac{21}{4}+\frac{19}{4}+\frac{61}{8}$$
$$=\frac{42+38+61}{8}=\frac{141}{8}$$
$$=17\frac{5}{8}$$

Example (iv)

Simplify: $\frac{5}{7}-\frac{2}{7}$

Solution

$$\frac{5}{7}-\frac{2}{7}=\frac{5-2}{7}=\frac{3}{7}.$$

Example (v)

Simplify: $2\frac{2}{3}-3\frac{1}{6}+6\frac{3}{4}$

Solution

$$2\frac{2}{3}-3\frac{1}{6}+6\frac{3}{4}=\frac{8}{3}-\frac{19}{6}+\frac{27}{4}$$

$$= \frac{32 - 38 + 81}{12}$$

$$= \frac{75}{12} = 6\,\frac{1}{4}$$

(i) Multiplication of a fraction by a whole number

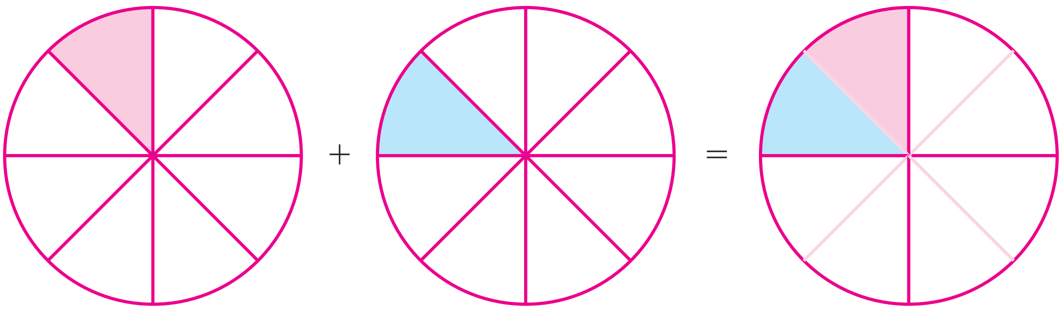


Fig. 1.1

Observe the pictures at the (fig.1.1) . Each shaded part is $\frac{1}{8}$ part of a circle. How much will the two shaded parts represent together?

They will represent $\frac{1}{8} + \frac{1}{8} = 2 \times \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$

To multiply a proper or improper fraction with the whole number:

we first multiply the whole number with the numerator of the fraction, keeping the denominator same. If the product is an improper fraction, convert it as a mixed fraction.

To multiply a mixed fraction by a whole number, first convert the mixed fraction to an improper fraction and then multiply.

Therefore, $4 \times 3\frac{4}{7} = 4 \times \frac{25}{7} = \frac{100}{7} = 14\frac{2}{7}$

Find :

i) $\frac{2}{5} \times 4$ ii) $\frac{8}{5} \times 4$

iii) $4 \times \frac{1}{5}$ iv) $\frac{13}{11} \times 6$

Find :

i) $6 \times 7\frac{2}{3}$

ii) $3\frac{2}{9} \times 7$

(ii) Fraction as an operator 'of'

From the figure (fig. 1.2) each shaded portion represents $\frac{1}{3}$ of 1. All the three shaded portions together will represent $\frac{1}{3}$ of 3.

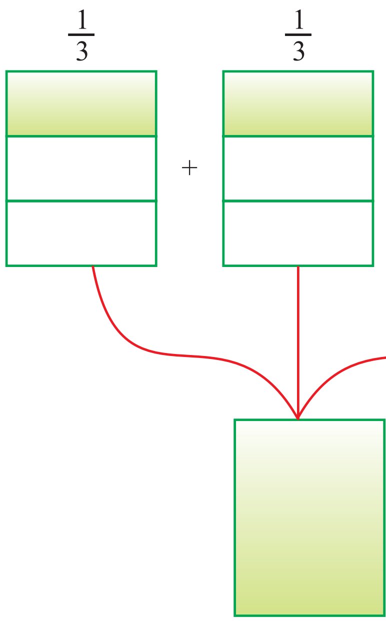


Fig. 1.2

Combining the 3 shaded portions we get 1.

Thus, one-third of 3 = $\frac{1}{3} \times 3 = 1$.

We can observe that 'of' represents multiplication.

Prema has 15 chocolates. Sheela has $\frac{1}{3}^{rd}$ of the number of chocolates what Prema has. How many chocolates Sheela has?

As, 'of' indicates multiplication, Sheela has $\frac{1}{3} \times 15 = 5$ chocolates.

Example 1.9

Find : $\frac{1}{4}$ of $2\frac{1}{5}$

Solution

$$\frac{1}{4} \text{ of } 2\frac{1}{5} = \frac{1}{4} \times 2\frac{1}{5}$$
$$= \frac{1}{4} \times \frac{11}{5}$$
$$= \frac{11}{20}$$

Example 1.10

In a group of 60 students $\frac{3}{10}$ of the total number of students like to study Science, $\frac{3}{5}$ of the total number like to study Social Science.

(i) How many students like to study Science?

(ii) How many students like to study Social Science?

Solution

Total number of students in the class = 60

(i) Out of 60 students, $\frac{3}{10}$ of the students like to study Science.

Thus, the number of students who like to study Science = $\frac{3}{10}$ of 60

$= \frac{3}{10} \times 60 = 18.$

(ii) Out of 60 students, $\frac{3}{5}$ of the students like to study Social Science.

Thus, the number of students who like to study Social Science

$= \frac{3}{5}$ of 60

$= \frac{3}{5} \times 60 = 36.$

Exercise 1.3

1. Multiply :

i) $6 \times \frac{4}{5}$ ii) $3 \times \frac{3}{7}$ iii) $4 \times \frac{4}{8}$ iv) $15 \times \frac{2}{10}$

v) $\frac{2}{3} \times 7$ vi) $\frac{5}{2} \times 8$ vii) $\frac{11}{4} \times 7$ viii) $\frac{5}{6} \times 12$

ix) $\frac{4}{7} \times 14$ x) $18 \times \frac{4}{3}$

2. Find :

i) $\frac{1}{2}$ of 28 ii) $\frac{7}{3}$ of 27 iii) $\frac{1}{4}$ of 64 iv) $\frac{1}{5}$ of 125

v) $\frac{8}{6}$ of 216 vi) $\frac{4}{8}$ of 32 vii) $\frac{3}{9}$ of 27 viii) $\frac{7}{10}$ of 100

ix) $\frac{5}{7}$ of 35 x) $\frac{1}{2}$ of 100

3. Multiply and express as a mixed fraction :

i) $5 \times 5\frac{1}{4}$ ii) $3 \times 6\frac{3}{5}$ iii) $8 \times 1\frac{1}{5}$ iv) $6 \times 10\frac{5}{7}$

v) $7 \times 7\frac{1}{2}$ vi) $9 \times 9\frac{1}{2}$

4. Vasu and Visu went for a picnic. Their mother gave them a baggage of 10 one litre water bottles. Vasu consumed $\frac{2}{5}$ of the water Visu consumed the remaining water. How much water did Vasu drink?

(iii) Multiplication of a fraction by a fraction

Example 1.11

Find $\frac{1}{5}$ of $\frac{3}{8}$.

Solution

$$\frac{1}{5} \text{ of } \frac{3}{8} = \frac{1}{5} \times \frac{3}{8} = \frac{3}{40}$$

Example 1.12

Find $\frac{2}{9} \times \frac{3}{2}$.

Solution

$$\frac{2}{9} \times \frac{3}{2} = \frac{1}{3}$$

Example 1.13

Leela reads $\frac{1}{4}^{th}$ of a book in 1 hour. How much of the book will she read in $3\frac{1}{2}$ hours?

Solution

The part of the book read by leela in 1 hour = $\frac{1}{4}$

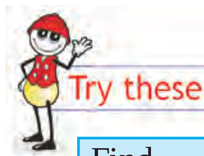
So, the part of the book read by her in $3\frac{1}{2}$ hour $= 3\frac{1}{2} \times \frac{1}{4}$

$$= \frac{7}{2} \times \frac{1}{4}$$

$$= \frac{7 \times 1}{4 \times 2}$$

$$= \frac{7}{8}$$

$\therefore$ Leela reads $\frac{7}{8}$ part of a book in $3\frac{1}{2}$ hours.



Find

i) $\frac{1}{3} \times \frac{7}{5}$

ii) $\frac{2}{3} \times \frac{8}{9}$

Exercise 1.4

1. Find :

i) $\frac{10}{5}$ of $\frac{5}{10}$ ii) $\frac{2}{3}$ of $\frac{7}{8}$ iii) $\frac{1}{3}$ of $\frac{7}{4}$ iv) $\frac{4}{8}$ of $\frac{7}{9}$

v) $\frac{4}{9}$ of $\frac{9}{4}$ vi) $\frac{1}{7}$ of $\frac{2}{9}$

2. Multiply and reduce to lowest form :

i) $\frac{2}{9} \times 3\frac{2}{3}$ ii) $\frac{2}{9} \times \frac{9}{10}$ iii) $\frac{3}{8} \times \frac{6}{9}$ iv) $\frac{7}{8} \times \frac{9}{14}$

v) $\frac{9}{2} \times \frac{3}{3}$ vi) $\frac{4}{5} \times \frac{12}{7}$

3. Simplify the following fractions :

i) $\frac{2}{5}\times 5\frac{2}{3}$ ii) $6\frac{3}{4}\times\frac{7}{10}$ iii) $7\frac{1}{2}\times 1$

iv) $5\frac{3}{4}\times 3\frac{1}{2}$ v) $7\frac{1}{4}\times 8\frac{1}{4}$

4. A car runs 20 km. using 1 litre of petrol. How much distance will it cover using $2\frac{3}{4}$ litres of petrol.

5. Everyday Gopal read book for $1\frac{3}{4}$ hours. He reads the entire book in 7 days. How many hours in all were required by him to read the book?

The reciprocal of a fraction

If the product of two non-zero numbers is equal to one then each number is called the reciprocal of the other. So reciprocal of $\frac{3}{5}$ is $\frac{5}{3}$, the reciprocal of $\frac{5}{3}$ is $\frac{3}{5}$.

Note: Reciprocal of 1 is 1 itself. 0 does not have a reciprocal.

(iv) Division of a whole number by a fraction

To divide a whole number by any fraction, multiply that whole number by the reciprocal of that fraction.

Example 1.14

Find (i) $6 \div \frac{2}{5}$ (ii) $8 \div \frac{7}{9}$

Solution

(i) $6 \div \frac{2}{5} = 6 \times \frac{5}{2} = 15$

(ii) $8 \div \frac{7}{9} = 8 \times \frac{9}{7} = \frac{72}{7}$

While dividing a whole number by a mixed fraction, first convert the mixed fraction into improper fraction and then solve it.

Try these

Find:
i) $6 \div 5\frac{2}{3}$ ii) $9 \div 3\frac{3}{7}$

Example 1.15

Find $6 \div 3\ \frac{4}{5}$

Solution

$6 \div 3\ \frac{4}{5} = 6 \div \frac{19}{5} = 6 \times \frac{5}{19} = \frac{30}{19} = 1\ \frac{11}{19}$

(v) Division of a fraction by another fraction

To divide a fraction by another fraction, multiply the first fraction by the reciprocal of the second fraction.

We can now find $\frac{1}{5} \div \frac{3}{7}$

$$\frac{1}{5} \div \frac{3}{7} = \frac{1}{5} \times \text{reciprocal of } \frac{3}{7}.$$

$$= \frac{1}{5} \times \frac{7}{3} = \frac{7}{15}$$

Try these

Find:

i) $\frac{3}{7} \div \frac{4}{5}$, ii) $\frac{1}{2} \div \frac{4}{5}$, iii) $2\frac{3}{4} \div \frac{7}{2}$

Exercise 1.5

1. Find the reciprocal of each of the following fractions:

   i) $\frac{5}{7}$ ii) $\frac{4}{9}$ iii) $\frac{10}{7}$ iv) $\frac{9}{4}$

   v) $\frac{33}{2}$ vi) $\frac{1}{9}$ vii) $\frac{1}{13}$ viii) $\frac{7}{5}$

2. Find :

   i) $\frac{5}{3} \div 25$ ii) $\frac{6}{9} \div 36$ iii) $\frac{7}{3} \div 14$ iv) $1\frac{1}{4} \div 15$

3. Find :

   (i) $\frac{2}{5} \div \frac{1}{4}$ (ii) $\frac{5}{6} \div \frac{6}{7}$ (iii) $2\frac{3}{4} \div \frac{3}{5}$ (iv) $3\frac{3}{2} \div \frac{8}{3}$

4. How many uniforms can be stitched from $47\frac{1}{4}$ metres of cloth if each scout requires $2\frac{1}{4}$ metres for one uniform?

5. The distance between two places is $47\frac{1}{2}$ km. If it takes $1\frac{3}{16}$ hours to cover the distance by a van, what is the speed of the van?

1.5 Introduction to Rational Numbers

A rational number is defined as a number that can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$. Here p is the numerator and q is the denominator.

For example $\frac{7}{3}, \frac{-5}{7}, \frac{2}{9}, \frac{11}{-7}, \frac{-3}{11}$ are the rational numbers

A rational number is said to be in *standard form* if its denominator is positive and the numerator and denominator have no common factor other than 1.

If a rational number is not in the standard form, then it can be reduced to the standard form.

Example 1.16

Reduce $\frac{72}{54}$ to the standard form.

Solution

We have, $\frac{72}{54} = \frac{72 \div 2}{54 \div 2}$

$$= \frac{36}{27} = \frac{36 \div 3}{27 \div 3}$$

$$= \frac{12}{9} = \frac{12 \div 3}{9 \div 3}$$

$$= \frac{4}{3}$$

Aliter: $\frac{72}{54} = \frac{72 \div 18}{54 \div 18} = \frac{4}{3}$

In this example, note that 18 is the highest common factor (H.C.F.) of 72 and 54.

To reduce the rational number to its standard form, we divide its numerator and denominator by their H.C.F. ignoring the negative sign if any.

If there is negative sign in the denominator divide by " – H.C.F.".

Example 1.17

Reduce to the standard form.

(i) $\frac{18}{-12}$ (ii) $\frac{-4}{-16}$

Write in standard form.

i) $\frac{-18}{51}$, ii) $\frac{-12}{28}$, iii) $\frac{7}{35}$

Solution

(i) The H.C.F. of 18 and 12 is 6

Thus, its standard form would be obtained by dividing by – 6.

$$\frac{18}{-12} = \frac{18 \div (-6)}{-12 \div (-6)} = \frac{-3}{2}$$

(ii) The H.C.F. of 4 and 16 is 4.

Thus, its standard form would be obtained by dividing by – 4

$$\frac{-4}{-16} = \frac{-4 \div (-4)}{-16 \div (-4)} = \frac{1}{4}$$

1.6 Representation of Rational numbers on the Number line.

You know how to represent integers on the number line. Let us draw one such number line.

The points to the right of 0 are positive integers. The points to left of 0 are negative integers.

Let us see how the rational numbers can be represented on a number line.

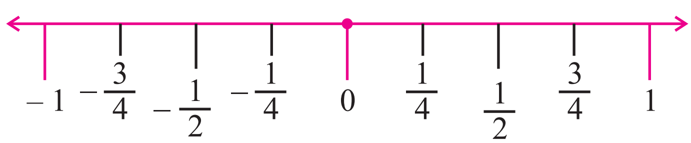


Fig. 1.3

Let us represent the number $-\frac{1}{4}$ on the number line.

As done in the case of positive integers, the positive rational numbers would be marked on the right of 0 and the negative rational numbers would be marked on the left of 0.

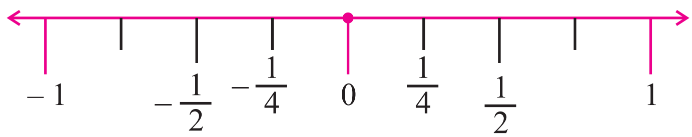


Fig. 1.4

To which side of 0, will you mark $-\frac{1}{4}$? Being a negative rational number, it would be marked to the left of 0.

You know that while marking integers on the number line, successive integers are marked at equal intervals. Also, from 0, the pair 1 and -1 is equidistant .

In the same way, the rational numbers $\frac{1}{4}$ and $-\frac{1}{4}$ would be at equal distance from 0. How to mark the rational number $\frac{1}{4}$? It is marked at a point which is one fourth of the distance from 0 to 1. So, $-\frac{1}{4}$ would be marked at a point which is one fourth of the distance from 0 to -1.

We know how to mark $\frac{3}{2}$ on the number line. It is marked on the right of 0 and lies halfway between 1 and 2. Let us now mark $\frac{-3}{2}$ on the number line. It lies on the left of 0 and is at the same distance as $\frac{3}{2}$ from 0.

Similarly $-\frac{1}{2}$ is to the left of zero and at the same distance from zero as $\frac{1}{2}$ is to the right. So as done above, $-\frac{1}{2}$ can be represented on the number line. All other rational numbers can be represented in a similar way.

Rational numbers between two rational numbers

Raju wants to count the whole numbers between 4 and 12. He knew there would be exactly 7 whole numbers between 4 and 12.

Are there any integers between 5 and 6 ?

There is no integer between 5 and 6.

$\therefore$ Number of integers between any two integers is finite.

Now let us see what will happen in the case of rational numbers ?

Raju wants to count the rational numbers between $\frac{3}{7}$ and $\frac{2}{3}$.

For that he converted them to rational numbers with same denominators.

So $\frac{3}{7} = \frac{9}{21}$ and $\frac{2}{3} = \frac{14}{21}$

Now he has, $\frac{9}{21} < \frac{10}{21} < \frac{11}{21} < \frac{12}{21} < \frac{13}{21} < \frac{14}{21}$

So $\frac{10}{21}, \frac{11}{21}, \frac{12}{21}, \frac{13}{21}$ are the rational numbers in between $\frac{9}{21}$ and $\frac{14}{21}$.

Now we can try to find some more rational numbers in between $\frac{3}{7}$ and $\frac{2}{3}$.

we have $\frac{3}{7} = \frac{18}{42}$ and $\frac{2}{3} = \frac{28}{42}$

So, $\frac{18}{42} < \frac{19}{42} < \frac{20}{42} < \cdots < \frac{28}{42}$. Therefore $\frac{3}{7} < \frac{19}{42} < \frac{20}{42} < \frac{21}{42} < \cdots < \frac{2}{3}$.

Hence we can find some more rational numbers in between $\frac{3}{7}$ and $\frac{2}{3}$.

We can find unlimited (infinite) number of rational numbers between any two rational numbers.

Example 1.18

List five rational numbers between $\frac{2}{5}$ and $\frac{4}{7}$.

Solution

Let us first write the given rational numbers with the same denominators.

Now, $\frac{2}{5} = \frac{2 \times 7}{5 \times 7} = \frac{14}{35}$ and $\frac{4}{7} = \frac{4 \times 5}{7 \times 5} = \frac{20}{35}$

So, we have $\frac{14}{35} < \frac{15}{35} < \frac{16}{35} < \frac{17}{35} < \frac{18}{35} < \frac{19}{35} < \frac{20}{35}$

$\frac{15}{35}, \frac{16}{35}, \frac{17}{35}, \frac{18}{35}, \frac{19}{35}$ are the five required rational numbers.

Example 1.19

Find seven rational numbers between $-\frac{5}{3}$ and $-\frac{8}{7}$.

Solution

Let us first write the given rational numbers with the same denominators.

Now, $-\frac{5}{3} = -\frac{5 \times 7}{3 \times 7} = -\frac{35}{21}$ and $-\frac{8}{7} = -\frac{8 \times 3}{7 \times 3} = -\frac{24}{21}$

So, we have $\frac{-35}{21} < \frac{-34}{21} < \frac{-33}{21} < \frac{-32}{21} < \frac{-31}{21} < \frac{-30}{21}$

$< -\frac{29}{21} < -\frac{28}{21} < -\frac{27}{21} < -\frac{26}{21} < -\frac{25}{21} < -\frac{24}{21}$

$\therefore$ The seven rational numbers are $-\frac{34}{21}, -\frac{33}{21}, -\frac{32}{21}, -\frac{31}{21}, -\frac{30}{21}, -\frac{29}{21}, -\frac{28}{21}$.

(We can take any seven rational numbers)

Exercise 1.6

1. Choose the best answer :

   i) $\frac{3}{8}$ is called a

   (A) positive rational number (B) negative rational number
   (C) whole number (D) positive integer

   ii) The proper negative rational number is

   (A) $\frac{4}{3}$ (B) $\frac{-7}{-5}$ (C) $-\frac{10}{9}$ (D) $\frac{10}{9}$

   iii) Which is in the standard form?

   (A) $-\frac{4}{12}$ (B) $-\frac{1}{12}$ (C) $\frac{1}{-12}$ (D) $\frac{-7}{14}$

   iv) A fraction is a

   (A) whole number (B) natural number
   (C) odd number (D) rational number

2. List four rational numbers between:

   i) $-\frac{7}{5}$ and $-\frac{2}{3}$ ii) $\frac{1}{2}$ and $\frac{4}{3}$ iii) $\frac{7}{4}$ and $\frac{8}{7}$

3. Reduce to the standard form:

   i) $\frac{-12}{16}$ ii) $\frac{-18}{48}$ iii) $\frac{21}{-35}$
   iv) $\frac{-70}{42}$ v) $\frac{-4}{8}$

4. Draw a number line and represent the following rational numbers on it.

   i) $\frac{3}{4}$ ii) $\frac{-5}{8}$ iii) $\frac{-8}{3}$
   iv) $\frac{6}{5}$ v) $-\frac{7}{10}$

5. Which of the following are in the standard form:

   i) $\frac{2}{3}$ ii) $\frac{4}{16}$ iii) $\frac{9}{6}$
   iv) $\frac{-1}{7}$ v) $\frac{-4}{7}$

1.7 Four Basic Operations on Rational numbers

You know how to add, subtract, multiply and divide on integers. Let us now study these four basic operations on rational numbers.

(i) Addition of rational numbers

Let us add two rational numbers with same denominator.

Example 1.20

Add $\frac{9}{5}$ and $\frac{7}{5}$.

Solution

$$\frac{9}{5}+\frac{7}{5} = \frac{9+7}{5} = \frac{16}{5}.$$

Let us add two rational numbers with different denominators.

Example 1.21

Simplify: $\frac{7}{3}+\left(\frac{-5}{4}\right)$

Solution

$$\frac{7}{3}+\left(\frac{-5}{4}\right)$$

$$= \frac{28-15}{12} \quad \text{(L.C.M. of 3 and 4 is 12)}$$

$$= \frac{13}{12}$$

Example 1.22

Simplify : $\frac{-3}{4}+\frac{1}{2}-\frac{5}{6}$.

Solution

$$\frac{-3}{4}+\frac{1}{2}-\frac{5}{6} = \frac{(-3\times 3)+(1\times 6)-(5\times 2)}{12} \quad \text{(L.C.M. of 4,2 and 6 is 12)}$$

$$= \frac{-9+6-10}{12}$$

$$= \frac{-19+6}{12} = \frac{-13}{12}$$

(ii) Subtraction of rational numbers

Example 1.23

Subtract : $\frac{8}{7}$ from $\frac{10}{3}$

Solution:

$$\frac{10}{3}-\frac{8}{7} = \frac{70-24}{21} = \frac{46}{21}$$

Example 1.24

Simplify $\frac{6}{35}-\left(\frac{-10}{35}\right)$

Solution:

$$\frac{6}{35}-\left(\frac{-10}{35}\right) = \frac{6+10}{35} = \frac{16}{35}$$

Example 1.25

Simplify : $\left(-2\frac{7}{35}\right)-\left(3\frac{6}{35}\right)$

Solution

$$\left(-2\frac{7}{35}\right)-\left(3\frac{6}{35}\right) = \frac{-77}{35}-\frac{111}{35}$$

$$= \frac{-77-111}{35} = \frac{-188}{35} = -5\frac{13}{35}$$

Example 1.26

The sum of two rational numbers is 1. If one of the numbers is $\frac{5}{20}$, find the other.

Solution

Sum of two rational numbers = 1

Given number + Required number = 1

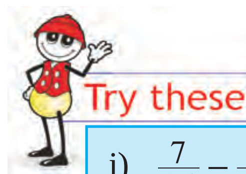
$$\frac{5}{20} + \text{Required number} = 1$$

$$\text{Required number} = 1-\frac{5}{20}$$

$$= \frac{20-5}{20}$$

$$= \frac{15}{20} = \frac{3}{4}$$

$\therefore$ Required number is $\frac{3}{4}$.



i) $\frac{7}{35}-\frac{5}{35}$, ii) $\frac{5}{6}-\frac{7}{12}$,

iii) $\frac{7}{3}-\frac{3}{4}$, iv) $\left(3\frac{3}{4}\right)-\left(2\frac{1}{4}\right)$,

v) $\left(4\frac{5}{7}\right)-\left(6\frac{1}{4}\right)$

Exercise1.7

1. Choose the best answer :

i) $\frac{1}{3}+\frac{2}{3}$ is equal to

(A) 2 (B) 3 (C) 1 (D) 4

ii) $\frac{4}{5}-\frac{9}{5}$ is equal to

(A) 1 (B) 3 (C) – 1 (D) 7

iii) $5\frac{1}{11}+1\frac{10}{11}$ is equal to

(A) 4 (B) 3 (C) – 5 (D) 7

iv) The sum of two rational numbers is 1. If one of the numbers is $\frac{1}{2}$, the other number is

(A) $\frac{4}{3}$ (B) $\frac{3}{4}$ (C) $\frac{-3}{4}$ (D) $\frac{1}{2}$

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2. Add :

i) $\frac{12}{5}$ and $\frac{6}{5}$ ii) $\frac{7}{13}$ and $\frac{17}{13}$ iii) $\frac{8}{7}$ and $\frac{6}{7}$

iv) $-\frac{7}{13}$ and $-\frac{5}{13}$ v) $\frac{7}{3}$ and $\frac{8}{4}$ vi) $-\frac{5}{7}$ and $\frac{7}{6}$

vii) $\frac{9}{7}$ and $-\frac{10}{3}$ viii) $\frac{3}{6}$ and $-\frac{7}{2}$ ix) $\frac{9}{4}, \frac{8}{7}$ and $\frac{1}{28}$

x) $\frac{4}{5}, -\frac{7}{10}$ and $-\frac{8}{15}$

3. Find the sum of the following :

i) $-\frac{3}{4}+\frac{7}{4}$ ii) $\frac{9}{6}+\frac{15}{6}$ iii) $-\frac{3}{4}+\frac{6}{11}$

iv) $-\frac{7}{8}+\frac{9}{16}$ v) $\frac{4}{5}+\frac{7}{20}$ vi) $\left(-\frac{6}{13}\right)+\left(-\frac{14}{26}\right)$

vii) $\frac{11}{13}+\left(-\frac{7}{2}\right)$ viii) $\left(-\frac{2}{5}\right)+\frac{5}{12}+\left(-\frac{7}{10}\right)$

ix) $\frac{7}{9}+\left(-\frac{10}{18}\right)+\left(-\frac{7}{27}\right)$ x) $\frac{6}{3}+\left(-\frac{7}{6}\right)+\left(-\frac{9}{12}\right)$

4. Simplify :

i) $\frac{7}{35}-\frac{5}{35}$ ii) $\frac{5}{6}-\frac{7}{12}$ iii) $\frac{7}{3}-\frac{3}{4}$

iv) $\left(3\frac{3}{4}\right)-\left(2\frac{1}{4}\right)$ v) $\left(4\frac{5}{7}\right)-\left(6\frac{1}{4}\right)$

5. Simplify :

i) $\left(1\frac{2}{11}\right)+\left(3\frac{5}{11}\right)$ ii) $\left(3\frac{4}{5}\right)-\left(7\frac{3}{10}\right)$

iii) $\left(-1\frac{2}{11}\right)+\left(-3\frac{5}{11}\right)+\left(6\frac{3}{11}\right)$ iv) $\left(-3\frac{9}{10}\right)+\left(3\frac{2}{5}\right)+\left(6\frac{5}{20}\right)$

v) $\left(-3\frac{4}{5}\right)+\left(2\frac{3}{8}\right)$ vi) $\left(-1\frac{5}{12}\right)+\left(-2\frac{7}{11}\right)$

vii) $\left(9\frac{6}{7}\right)+\left(-11\frac{2}{3}\right)+\left(-5\frac{7}{42}\right)$ viii) $\left(7\frac{3}{10}\right)+\left(-10\frac{7}{21}\right)$

6. The sum of two rational numbers is $\frac{17}{4}$. If one of the numbers is $\frac{5}{2}$, find the other number.

7. What number should be added to $\frac{5}{6}$ so as to get $\frac{49}{30}$.

8. A shopkeeper sold $7\frac{3}{4}$kg, $2\frac{1}{2}$kg and $3\frac{3}{5}$kg of sugar to three consumers in a day. Find the total weight of sugar sold on that day.

9. Raja bought 25 kg of Rice and he used $1\frac{3}{4}$ kg on the first day, $4\frac{1}{2}$ kg on the second day. Find the remaining quantity of rice left.

10. Ram bought 10 kg apples and he gave $3\frac{4}{5}$kg to his sister and $2\frac{3}{10}$ kg to his friend. How many kilograms of apples are left?

(iii) Multiplication of Rational numbers

To find the product of two rational numbers, multiply the numerators and multiply the denominators separately and put them as new rational number. Simplify the new rational number into its lowest form.

Example 1.27

Find the product of $\left(\frac{4}{-11}\right)$ and $\left(\frac{-22}{8}\right)$.

Solution

$$\left(\frac{4}{-11}\right)\times\left(\frac{-22}{8}\right)$$

$$=\left(\frac{-4}{11}\right)\times\left(\frac{-22}{8}\right) \quad = \frac{88}{88}$$

$$= 1$$

Example 1.28

Find the product of $\left(-2\frac{4}{15}\right)$ and $\left(-3\frac{2}{49}\right)$.

Solution

$$\left(-2\frac{4}{15}\right)\times\left(-3\frac{2}{49}\right) = \left(\frac{-34}{15}\right)\times\left(\frac{-149}{49}\right)$$

$$= \frac{5066}{735} = 6\frac{656}{735}$$

Example 1.29

The product of two rational numbers is $\frac{2}{9}$. If one of the numbers is $\frac{1}{2}$, find the other rational number.

Solution

The product of two rational numbers $= \frac{2}{9}$

One rational number $= \frac{1}{2}$

$\therefore$ Given number $\times$ required number $= \frac{2}{9}$

$\frac{1}{2}\times$ required number $= \frac{2}{9}$

required number $= \frac{2}{9}\times\frac{2}{1} = \frac{4}{9}$

$\therefore$ Required rational number is $\frac{4}{9}$.

Multiplicative inverse (or reciprocal) of a rational number

If the product of two rational numbers is equal to 1, then one number is called the multiplicative inverse of other.

i) $\frac{7}{23} \times \frac{23}{7} = 1$

$\therefore$ The multiplicative inverse of $\frac{7}{23}$ is $\frac{23}{7}$.

Similarly the multiplicative inverse of $\frac{23}{7}$ is $\frac{7}{23}$.

ii) $\left(\frac{-8}{12}\right) \times \left(\frac{12}{-8}\right) = 1$

$\therefore$ The multiplicative inverse of $\left(\frac{-8}{12}\right)$ is $\left(\frac{12}{-8}\right)$.

Find

1) $\frac{7}{8} \times \frac{9}{12}$, 2) $\frac{11}{12} \times \frac{24}{33}$

3) $\left(-1\frac{1}{4}\right) \times \left(-7\frac{2}{3}\right)$

(iv) Division of rational numbers

To divide one rational number by another rational number, multiply the first rational number with the multiplicative inverse of the second rational number.

Example 1.30

Find $\left(\frac{2}{3}\right) \div \left(\frac{-5}{10}\right)$.

Solution

$$\left(\frac{2}{3}\right) \div \left(\frac{-5}{10}\right) = \frac{2}{3} \div \left(\frac{-1}{2}\right)$$
$$= \frac{2}{3} \times (-2) = \frac{-4}{3}$$

Example 1.31

Find $4\frac{3}{7} \div 2\frac{3}{8}$.

Solution

$$4\frac{3}{7} \div 2\frac{3}{8} = \frac{31}{7} \div \frac{19}{8}$$
$$= \frac{31}{7} \times \frac{8}{19} = \frac{248}{133}$$
$$= 1\frac{115}{133}$$

Exercise 1.8

1. Choose the best answer :

i) $\frac{7}{13} \times \frac{13}{7}$ is equal to

(A) 7 (B) 13 (C) 1 (D) – 1

ii) The multiplicative inverse of $\frac{7}{8}$ is

(A) $\frac{7}{8}$ (B) $\frac{8}{7}$ (C) $\frac{-7}{8}$ (D) $\frac{-8}{7}$

iii) $\frac{4}{-11} \times \left(\frac{-22}{8}\right)$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4

iv) $-\frac{4}{9} \div \frac{9}{36}$ is equal to

(A) $\frac{-16}{9}$ (B) 4 (C) 5 (D) 7

2. Multiply :

i) $\frac{-12}{5}$ and $\frac{6}{5}$ ii) $\frac{-7}{13}$ and $\frac{5}{13}$

iii) $\frac{-3}{9}$ and $\frac{7}{8}$ iv) $\frac{-6}{11}$ and $\frac{44}{22}$

v) $\frac{-50}{7}$ and $\frac{28}{10}$ vi) $\frac{-5}{6}$ and $\frac{-4}{15}$

3. Find the value of the following :

i) $\frac{9}{5} \times \frac{-10}{4} \times \frac{15}{18}$ ii) $\frac{-8}{4} \times \frac{-5}{6} \times \frac{-30}{10}$

iii) $1\frac{1}{5} \times 2\frac{2}{5} \times 9\frac{3}{10}$ iv) $-3\frac{4}{15} \times -2\frac{1}{5} \times 9\frac{1}{5}$ v) $\frac{3}{6} \times \frac{9}{7} \times \frac{10}{4}$

4. Find the value of the following :

i) $\frac{-4}{9} \div \frac{9}{-4}$ ii) $\frac{3}{5} \div \left(\frac{-4}{10}\right)$

iii) $\left(\frac{-8}{35}\right) \div \frac{7}{35}$ iv) $-9\frac{3}{4} \div 1\frac{3}{40}$

5. The product of two rational numbers is 6. If one of the number is $\frac{14}{3}$, find the other number.

6. What number should be multiply $\frac{7}{2}$ to get $\frac{21}{4}$?

1.8 Decimal numbers

(i) Represent Rational Numbers as Decimal numbers

You have learnt about decimal numbers in the earlier classes. Let us briefly recall them here.

All rational numbers can be converted into decimal numbers.

For Example

(i) $\frac{1}{8} = 1 \div 8$

$\therefore \frac{1}{8} = 0.125$

(ii) $\frac{3}{4} = 3 \div 4$

$\therefore \frac{3}{4} = 0.75$

(iii) $3\frac{1}{5} = \frac{16}{5} = 3.2$

(iv) $\frac{2}{3} = 0.6666\cdots$ Here 6 is recurring without end.

(ii) Addition and Subtraction of decimals

Example 1.32

Add 120.4, 2.563, 18.964

Solution

$$\begin{array}{r} 120.4 \\ 2.563 \\ 18.964 \\ \hline 141.927 \\ \hline \end{array}$$

Example 1.33

Subtract 43.508 from 63.7

Solution

$$\begin{array}{rr} & 63.700 \\ (-) & 43.508 \\ \hline & 20.192 \\ \hline \end{array}$$

Example 1.34

Find the value of 27.69 – 14.04 + 35.072 – 10.12.

Solution

$$\begin{array}{r} 27.690 \\ 35.072 \\ \hline 62.762 \\ \hline \end{array} \qquad \begin{array}{r} -14.04 \\ -10.12 \\ \hline -24.16 \\ \hline \end{array} \qquad \begin{array}{r} 62.762 \\ -24.16 \\ \hline 38.602 \\ \hline \end{array}$$

The value is 38.602.

Examples 1.35

Deepa bought a pen for ₹177.50. a pencil for ₹4.75 and a notebook for ₹20.60. What is her total expenditure?

Solution

Cost of one pen = ₹177.50

Cost of one pencil = ₹4.75

Cost of one notebook = ₹20.60

∴ Deepa's total expenditure = ₹202.85

(iii) Multiplication of Decimal Numbers

Rani purchased 2.5 kg fruits at the rate of ₹23.50 per kg. How much money should she pay? Certainly it would be ₹(2.5 × 23.50). Both 2.5 and 23.5 are decimal numbers. Now, we have come across a situation where we need to know how to multiply two decimals. So we now learn the multiplication of two decimal numbers.

Let us now find 1.5 × 4.3

Multiplying 15 and 43. We get 645. Both, in 1.5 and 4.3, there is 1 digit to the right of the decimal point. So, count 2 digits from the right and put a decimal point. (since 1 + 1 = 2)

While multiplying 1.43 and 2.1, you will first multiply 143 and 21. For placing the decimal in the product obtained, you will count 2 + 1 = 3 digits starting from the right most digit. Thus 1.43 × 2.1 = 3.003.

Try these

i) 2.9 × 5
ii) 1.9 × 1.3
iii) 2.2 × 4.05

Example 1.36

The side of a square is 3.2 cm. Find its perimeter.

Do you know?

Perimeter of a square = 4 × side

Solution

All the sides of a square are equal.

Length of each side = 3.2 cm.

Perimeter of a square = 4 × side

Thus, perimeter = 4 × 3.2 = 12.8 cm.

Example 1.37

The length of a rectangle is 6.3 cm and its breath is 3.2 cm. What is the area of the rectangle?

Solution

Length of the rectangle = 6.3 cm

Breadth of the rectangle = 3.2 cm.

Area of the rectangle = (length) × (breath)

= 6.3 × 3.2 = 20.16 cm^2

Multiplication of Decimal number by 10, 100 and 1000

Rani observed that $3.7 = \frac{37}{10}$, $3.72 = \frac{372}{100}$ and $3.723 = \frac{3723}{1000}$ Thus, she found that depending on the position of the decimal point the decimal number can be converted to a fraction with denominator 10 , 100 or 1000. Now let us see what would happen if a decimal number is multiplied by 10 or 100 or 1000.

For example,

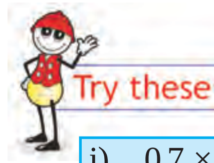
$$3.23 \times 10 = \frac{323}{100} \times 10 = 32.3$$

Decimal point shifted to the right by one place since 10 has one zero over one.

$$3.23 \times 100 = \frac{323}{100} \times 100 = 323$$

Decimal point shifted to the right by two places since 100 has two zeros over two.

$$3.23 \times 1000 = \frac{323}{100} \times 1000 = 3230$$



i) 0.7×10
ii) 1.3×100
iii) 76.3×1000

Exercise 1.9

1. Choose the best answer :
   i) 0.1×0.1 is equal to
   (A) 0.1 (B) 0.11 (C) 0.01 (D) 0.0001
   ii) $5 \div 100$ is equal to
   (A) 0.5 (B) 0.005 (C) 0.05 (D) 0.0005
   iii) $\frac{1}{10} \times \frac{1}{10}$ is equal to
   (A) 0.01 (B) 0.001 (C) 0.0001 (D) 0.1
   iv) 0.4×5 is equal to
   (A) 1 (B) 0.4 (C) 2 (D) 3

2. Find :
   (i) 0.3×7 (ii) 9×4.5 (iii) 2.85×6 (iv) 20.7×4
   (v) 0.05×9 (vi) 212.03×5 (vii) 3×0.86 (viii) 3.5×0.3
   (ix) 0.2×51.7 (x) 0.3×3.47 (xi) 1.4×3.2 (xii) 0.5×0.0025
   (xiii) 12.4×0.17 (xiv) 1.04×0.03

3. Find :
   (i) 1.4×10 (ii) 4.68×10 (iii) 456.7×10 (iv) 269.08×10
   (v) 32.3×100 (vi) 171.4×100 (vii) 4.78×100

4. Find the area of rectangle whose length is 10.3 cm and breath is 5 cm.

5. A two-wheeler covers a distance of 75.6 km in one litre of petrol. How much distance will it cover in 10 litres of petrol?

(iv) Division of Decimal Numbers

Jasmine was preparing a design to decorate her classroom. She needed a few colourd strips of paper of length 1.8 cm each. She had a strip of coloured paper of length 7.2 cm. How many pieces of the required length will she get out of this strip? She thought it would be $\frac{7.2}{1.8}$ cm. Is she correct?

Both 7.2 and 1.8 are decimal numbers. So we need to know the division of decimal numbers .

For example,

$$141.5 \div 10 = 14.15$$
$$141.5 \div 100 = 1.415$$
$$141.5 \div 1000 = 0.1415$$

To get the quotient we shift the point in the decimal number to the left by as many places as there are zeros over 1.

Try these

Find:

i) $432.5 \div 10$

ii) $432.5 \div 100$

iii) $432.5 \div 1000$

Example 1.38

Find $4.2 \div 3$.

Solution

$$4.2 \div 3 = \frac{42}{10} \div 3 = \frac{42}{10} \times \frac{1}{3}$$
$$= \frac{42 \times 1}{10 \times 3} = \frac{1 \times 42}{10 \times 3}$$
$$= \frac{1}{10} \times \frac{42}{3} = \frac{1}{10} \times 14$$
$$= \frac{14}{10} = 1.4$$

Try these

Find:

i) $85.8 \div 3$

ii) $25.5 \div 5$

Example 1.39

Find $18.5 \div 5$.

Solution

First find $185 \div 5$. We get 37.

There is one digit to the right of the decimal point in 18.5. Place the decimal point in 37 such that there would be one digit to its right. We will get 3.7.

Try these

Find:

i) $73.12 \div 4$

ii) $34.55 \div 7$

Division of a Decimal Number by another Decimal number

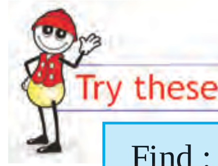
Example 1.40

Find $\frac{17.6}{0.4}$.

Solution

We have $17.6 \div 0.4 = \frac{176}{10} \div \frac{4}{10}$

$$= \frac{176}{10} \times \frac{10}{4} = 44.$$



Find :

i) $\frac{9.25}{0.5}$

ii) $\frac{36}{0.04}$

iii) $\frac{6.5}{1.3}$

Example 1.41

A car covers a distance of 129.92 km in 3.2 hours. What is the distance covered by it in 1 hour?

Solution

Distance covered by the car = 129.92 km.

Time required to cover this distance = 3.2 hours.

So, distance covered by it in 1 hour = $\frac{129.92}{3.2} = \frac{1299.2}{32}$ = 40.6km.

Exercise 1.10

1. Choose the best answer :

i) 0.1 ÷ 0.1 is equal to

(A) 1 (B) 0.1 (C) 0.01 (D) 2

ii) $\frac{1}{1000}$ is equal to

(A) 0.01 (B) 0.001 (C) 1.001 (D) 1.01

iii) How many apples can be bought for ₹50 if the cost of one apple is ₹12.50?

(A) 2 (B) 3 (C) 4 (D) 7

iv) $\frac{12.5}{2.5}$ is equal to

(A) 4 (B) 5 (C) 7 (D) 10

2. Find :

(i) 0.6 ÷ 2 (ii) 0.45 ÷ 5 (iii) 3.48 ÷ 3

(iv) 64.8 ÷ 6 (v) 785.2 ÷ 4 (vi) 21.28 ÷ 7

3. Find :

(i) 6.8 ÷ 10 (ii) 43.5 ÷ 10 (iii) 0.9 ÷ 10

(iv) 44.3 ÷ 10 (v) 373.48 ÷ 10 (vi) 0.79 ÷ 10

4. Find :

   (i) 5.6 ÷ 100 (ii) 0.7 ÷ 100 (iii) 0.69 ÷ 100

   (iv) 743.6 ÷ 100 (v) 43.7 ÷ 100 (vi) 78.73 ÷ 100

5. Find :

   (i) 8.9 ÷ 1000 (ii) 73.3 ÷ 1000 (iii) 48.73 ÷ 1000

   (iv) 178.9 ÷ 1000 (v) 0.9 ÷ 1000 (vi) 0.09 ÷ 1000

6. Find :

   (i) 9 ÷ 4.5 (ii) 48 ÷ 0.3 (iii) 6.25 ÷ 0.5

   (iv) 40.95 ÷ 5 (v) 0.7 ÷ 0.35 (vi) 8.75 ÷ 0.25

7. A vehicle covers a distance of 55.2 km in 2.4 litres of petrol. How much distance will it cover in one litre of petrol?
8. If the total weight of 11 similar bags is 115.5 kg, what is the weight of 1 bag?
9. How many books can be bought for ₹362.25, if the cost of one book is ₹40.25?
10. A motorist covers a distance of 135.04 km in 3.2 hours. Find his speed?
11. The product of two numbers is 45.36. One of them is 3.15. Find the other number?

1.9 Powers

Introduction

Teacher asked Ramu, “Can you read this number 2560000000000000?”

He replies, “It is very difficult to read sir”.

“The distance between sun and saturn is 1,433,500,000,000 m. Raja can you able to read this number?” asked teacher.

He replies, “Sir, it is also very difficult to read”.

Now, we are going to see how to read the difficult numbers in the examples given above.

Exponents

We can write the large numbers in a shortest form by using the following methods.

$$10 = 10^1$$

$$100 = 10^1 \times 10^1 = = 10^2$$

$$1000 = 10^1 \times 10^1 \times 10^1 = 10^3$$

Similarly,

$$2^1 \times 2^1 = 2^2$$

$$2^1 \times 2^1 \times 2^1 = 2^3$$

$$2^1 \times 2^1 \times 2^1 \times 2^1 = 2^4$$

$a \times a = a^2$ [read as 'a' squared or 'a' raised to the power 2]

$a \times a \times a = a^3$ [read as 'a' cubed or 'a' raised to the power 3]

$a \times a \times a \times a = a^4$ [read as 'a' raised to the power 4 or the 4th power of 'a']

....

....

$a \times a \times ...\ m$ times $= a^m$ [read as 'a' raised to the power m or mth power of 'a']

Here 'a' is called the base, 'm' is called the exponent (or) power.

Note: Only a^2 and a^3 have the special names "a squared' and "a cubed".

∴ we can write large numbers in a shorter form using exponents.

Example 1.42

Express 512 as a power .

Solution

We have $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

So we can say that $512 = 2^9$

Example: 1.43

Which one is greater 2^5 , 5^2 ?

Solution

We have $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

and $5^2 = 5 \times 5 = 25$

Since $32 > 25$.

Therefore 2^5 is greater than 5^2.

Example: 1.44

Express the number 144 as a product of powers of prime factors.

Solution

$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$= 2^4 \times 3^2$$
$$\text{Thus, } 144 = 2^4 \times 3^2$$

Example 1.45

Find the value of (i) 4^5 (ii) $(-4)^5$

Solution

(i) 4^5

$$4^5 = 4 \times 4 \times 4 \times 4 \times 4$$
$$= 1024.$$

(ii) $(-4)^5$

$$(-4)^5 = (-4) \times (-4) \times (-4) \times (-4) \times (-4)$$
$$= -1024.$$

Excercise 1.11

1. Choose the best answer :

   i) -10^2 is equal to

   (A) – 100 (B) 100 (C) – 10 (D) 10

   ii) $(-10)^2$ is equal to

   (A) 100 (B) – 100 (C) 10 (D) – 10

   iii) $a \times a \times a \times .....\ n$ times is equal to

   (A) a^m (B) a^{-n} (C) a^n (D) a^{m+n}

   iv) $103^3 \times 0$ is equal to

   (A) 103 (B) 9 (C) 0 (D) 3

2. Find the value of the following :

   (i) 2^8 (ii) 3^3 (iii) 11^3

   (iv) 12^3 (v) 13^4 (vi) 0^{10}

3. Express the following in exponential form :

   (i) $7 \times 7 \times 7 \times 7 \times 7 \times 7$ (ii) $1 \times 1 \times 1 \times 1 \times 1$

   (iii) $10 \times 10 \times 10 \times 10 \times 10 \times 10$ (iv) $b \times b \times b \times b \times b$

   (v) $2 \times 2 \times a \times a \times a \times a$ (vi) $1003 \times 1003 \times 1003$

4. Express each of the follwing numbers using exponential notation. (with smallest base)

   (i) 216 (ii) 243 (iii) 625
   (iv) 1024 (v) 3125 (vi) 100000

5. Identify the greater number in each of the following :

   (i) 4^5 , 5^4 (ii) 2^6 , 6^2 (iii) 3^2 , 2^3
   (iv) 5^6 , 6^5 (v) 7^2 , 2^7 (vi) 4^7 , 7^4

6. Express each of the following as product of powers of their prime factors :

   (i) 100 (ii) 384 (iii) 798
   (iv) 678 (v) 948 (vi) 640

7. Simplify :

   (i) 2×10^5 (ii) 0×10^4 (iii) $5^2 \times 3^4$
   (iv) $2^4 \times 3^4$ (v) $3^2 \times 10^9$ (vi) $10^3 \times 0$

8. Simplify :

   (i) $(-5)^3$ (ii) $(-1)^{10}$ (iii) $(-3)^2 \times (-2)^3$
   (iv) $(-4)^2 \times (-5)^3$ (v) $(6)^3 \times (7)^2$ (vi) $(-2)^7 \times (-2)^{10}$

Laws of exponents

Multiplying powers with same base

1) $3^2 \times 3^4 = (3 \times 3) \times (3 \times 3 \times 3 \times 3)$
$= 3^1 \times 3^1 \times 3^1 \times 3^1 \times 3^1 \times 3^1$
$= 3^6$

2) $(-5)^2 \times (-5)^3 = [(-5) \times (-5)] \times [(-5) \times (-5) \times (-5)]$
$= (-5)^1 \times (-5)^1 \times (-5)^1 \times (-5)^1 \times (-5)^1$
$= (-5)^5$

3) $a^2 \times a^5 = (a \times a) \times (a \times a \times a \times a \times a)$
$= a^1 \times a^1 \times a^1 \times a^1 \times a^1 \times a^1 \times a^1 = a^7$

From this we can generalise that for any non-zero integer a, where m and n are whole numbers $a^m \times a^n = a^{m+n}$

i) $2^5 \times 2^7$ ii) $4^3 \times 4^4$
iii) $p^3 \times p^5$ iv) $(-4)^{100} \times (-4)^{10}$

Dividing powers with the same base

We observe the following examples:

i) $2^7 \div 2^5 = \dfrac{2^7}{2^5}$

$= \dfrac{2\times2\times2\times2\times2\times2\times2}{2\times2\times2\times2\times2} = 2^2$

ii) $(-5)^4 \div (-5)^3 = \dfrac{(-5)^4}{(-5)^3}$

$= \dfrac{(-5)\times(-5)\times(-5)\times(-5)}{(-5)\times(-5)\times(-5)} = -5$

From these examples, we observe: In general, for any non-zero integer '*a*',

$a^m \div a^n = a^{m-n}$ where m and n are whole numbers and $m > n$. If $n = m$ $a^m \div a^m = a^{m-m} = a^0 = 1$.

Power of a power

Consider the following:

(i) $(3^3)^2 = 3^3 \times 3^3 = 3^{3+3} = 3^6$

(ii) $(2^2)^3 = 2^2 \times 2^2 \times 2^2 = 2^{2+2+2} = 2^6$

From this we can generalise for any non-zero integer '*a*'

$(a^m)^n = a^{mn}$, where m and n are whole numbers.

Example: 1.46

Write the exponential form for 9 × 9 × 9 × 9 by taking base as 3.

Solution

We have $9 \times 9 \times 9 \times 9 = 9^4$

We know that $9 = 3 \times 3$

Therefore $9^4 = (3^2)^4 = 3^8$

Exercise 1.12

1. Choose the best answer :

i) $a^m \times a^x$ is equal to

(A) a^{mx} (B) a^{m+x} (C) a^{m-x} (D) a^{m^x}

ii) $10^{12} \div 10^{10}$ is equal to

(A) 10^2 (B) 1 (C) 0 (D) 10^{10}

iii) $10^{10} \times 10^{2}$ is equal to

(A) 10^{5} (B) 10^{8} (C) 10^{12} (D) 10^{20}

iv) $(2^{2})^{10}$ is equal to

(A) 2^{5} (B) 2^{12} (C) 2^{20} (D) 2^{10}

Using laws of exponents, simplify in the exponential form.

2. i) $3^{5} \times 3^{3} \times 3^{4}$

ii) $a^{3} \times a^{2} \times a^{7}$

iii) $7^{x} \times 7^{2} \times 7^{3}$

iv) $10^{0} \times 10^{2} \times 10^{5}$

v) $5^{6} \times 5^{2} \times 5^{1}$

3. i) $5^{10} \div 5^{6}$

ii) $a^{6} \div a^{2}$

iii) $10^{10} \div 10^{0}$

iv) $4^{6} \div 4^{4}$

v) $3^{3} \div 3^{3}$

4. i) $(3^{4})^{3}$

ii) $(2^{5})^{4}$

iii) $(4^{5})^{2}$

iv) $(4^{0})^{10}$

v) $(5^{2})^{10}$

Activity

Multiplication of fractions pictorially

Step 1 :

Take a transparent sheet of paper.

Step 2 :

Draw a rectangle 16 cm by 10 cm and divide it vertically in to 8 equal parts. Shade the first 3 parts. The shaded portion represents 3/8 of the rectangle.

Step 3 :

Draw another rectangle of the same size and divide it horizontally into 5 equal parts. Shade the first 2 parts. The shaded portion represents 2/5 of the rectangle.

Step 4 :

Place the first transparent sheet on the top of the second sheet so that the two rectangles coincide.

We find that,

Total number of squares = 40

Number of squares shaded vertically and horizontally = 6

$$\therefore \frac{3}{8} \times \frac{2}{5} = \frac{6}{40}$$

Points to Remember

1. Natural numbrs N = {1, 2, 3, ...}
2. Whole numbers W = {0, 1, 2, ...}
3. Integers Z = {..., – 3, – 2, – 1, 0, 1, 2, 3, ...}
4. The product of two positive integers is a positive integer.
5. The product of two negative integers is a positive integer.
6. The product of a positive integer and a negative integer is a negative integer.
7. The division of two integers need not be an integer.
8. Fraction is a part of whole.
9. If the product of two non-zero numbers is 1 then the numbers are called the reciprocal of each other.
10. a × a × a × ... m times = a^m
    (read as 'a' raised to the power m (or) the m^{th} power of 'a')
11. For any two non-zero integers a and b and whole numbers m and n,
    i) $a^m a^n = a^{m+n}$
    ii) $\frac{a^m}{a^n} = a^{m-n}$, where m > n
    iii) $(a^m)^n = a^{mn}$
    iv) $(-1)^n = 1$, when n is an even number
    $(-1)^n = -1$, when n is an odd number

Chapter 2

2 ALGEBRA

2.1 ALGEBRAIC EXPRESSIONS

(i) Introduction

In class VI, we have already come across simple algebraic expressions like $x + 10$, $y - 9$, $3m + 4$, $2y - 8$ and so on.

Expression is a main concept in algebra. In this chapter you are going to learn about algebraic expressions, how they are formed, how they can be combined, how to find their values, and how to frame and solve simple equations.

(ii) Variables, Constants and Coefficients

Variable

A quantity which can take various numerical values is known as a **variable** (or a **literal**).

Variables can be denoted by using the letters a, b, c, x, y, z, etc.

Constant

A quantity which has a fixed numerical value is called a **constant**.

For example, 3, -25, $\frac{12}{13}$ and 8.9 are constants.

Numerical expression

A number or a combination of numbers formed by using the arithmetic operations is called a **numerical expression** or **an arithmetic expression**.

For example, $3 + (4 \times 5)$, $5 - (4 \times 2)$, $(7 \times 9) \div 5$ and $(3 \times 4) - (4 \times 5 - 7)$ are numerical expressions.

Algebraic Expression

An algebraic expression is a combination of variables and constants connected by arithmetic operations.

Example 2.1

	Statement	**Expressions**
(i)	5 added to y	$y + 5$
(ii)	8 subtracted from n	$n - 8$
(iii)	12 multiplied by x	$12\,x$
(iv)	p divided by 3	$\frac{p}{3}$

Term

A term is a constant or a variable or a product of a constant and one or more variables.

$3x^2$, $6x$ and -5 are called the terms of the expression $3x^2 + 6x - 5$.

A term could be

(i) a constant
(ii) a variable
(iii) a product of constant and a variable (or variables)
(iv) a product of two or more variables

In the expression $4a^2 + 7a + 3$, the terms are $4a^2$, $7a$ and 3. The number of terms is 3.

In the expression $-6p^2 + 18pq + 9q^2 - 7$, the terms are $-6p^2$, $18pq$, $9q^2$ and -7. The number of terms is 4.

Find the number of terms :

(i) $8b$
(ii) $3p - 2q$
(iii) $a^2 + 4a - 5$
(iv) $7x^2y - 4y + 8x - 9$
(v) $4m^2n + 3mn^2$

Coefficient

The coefficient of a given variable or factor in a term is another factor whose product with the given variable or factor is the term itself.

Do you know?

In the term $6xy$, the factors are $6, x, y, 6x, 6y, xy$ and $6xy$.

If the coefficient is a constant, it is called a constant coefficient or a numerical coefficient.

Example 2.2

In the term $5xy$,

coefficient of xy is 5 (numerical coefficient),

coefficient of $5x$ is y,

coefficient of $5y$ is x.

Find the numerical coefficient in

(i) $3z$ (ii) $8ax$ (iii) ab

(iv) $-pq$ (v) $\frac{1}{2}mn$ (vi) $-\frac{4}{7}yz$

Example 2.3

In the term $-mn^2$,

coefficient of mn^2 is -1,

coefficient of $-n^2$ is m,

coefficient of m is $-n^2$.

Activity

An algebraic box contains cards that have algebraic expressions written on it. Ask each student to pick out a card from the box and answer the following :

- Number of terms in the expression
- Coefficients of each term in the expression
- Constants in the expression

S.No.	Expression	Term which contains y	Coefficient of y
1.	$10-2y$		
2.	$11+yz$	yz	z
3.	yn^2+10		
4.	$-3m^2y+n$		

Exercise 2.1

1. Choose the correct answer:

   (i) The numerical coefficient in $-7xy$ is

   (A) -7 (B) x (C) y (D) xy

   (ii) The numerical coefficient in $-q$ is

   (A) q (B) $-q$ (C) 1 (D) -1

   (iii) 12 subtracted from z is

   (A) $12 + z$ (B) $12z$ (C) $12 - z$ (D) $z - 12$

   (iv) n multiplied by -7 is

   (A) $7n$ (B) $-7n$ (C) $\frac{7}{n}$ (D) $\frac{-7}{n}$

   (v) Three times p increased by 7 is

   (A) 21p (B) $3p - 7$ (C) $3p + 7$ (D) $7 - 3p$

2. Identify the constants and variables from the following:
   a, 5, $-xy$, p, -9.5

3. Rewrite each of the following as an algebraic expression

   (i) 6 more than x
   (ii) 7 subtracted from $-m$
   (iii) 11 added to 3 q
   (iv) 10 more than 3 times x
   (v) 8 less than 5 times y

4. Write the numerical coefficient of each term of the expression $3y^2 - 4yx + 9x^2$.

5. Identify the term which contains x and find the coefficient of x

   (i) $y^2x + y$ (ii) $3 + x + 3x^2y$

   (iii) $5 + z + zx$ (iv) $2x^2y - 5xy^2 + 7y^2$

6. Identify the term which contains y^2 and find the coefficient of y^2

   (i) $3 - my^2$ (ii) $6y^2 + 8x$ (iii) $2x^2y - 9xy^2 + 5x^2$

(iii) Power

If a variable a is multiplied five times by itself then it is written as $a \times a \times a \times a \times a = a^5$(read as a to the power 5). Similarly, $b \times b \times b = b^3$ (b to the power 3) and $c \times c \times c \times c = c^4$($c$ to the power 4). Here a, b, c are called the base and 5, 3, 4 are called the exponent or power.

Example 2.4

(i) In the term $-8a^2$, the power of the variable a is 2

(ii) In the term m, the power of the variable m is 1.

(iv) Like terms and Unlike terms

Terms having the same variable or product of variables with same powers are called **Like terms**. Terms having different variable or product of variables with different powers are called **Unlike terms**.

Example 2.5

(i) $x, \; -5x, \; 9x$ are like terms as they have the same variable x

(ii) $4x^2y, \; -7yx^2$ are like terms as they have the same variable x^2y

Example 2.6

(i) $6x, \; 6y$ are unlike terms

(ii) $3xy^2, \; 5xy, \; 8x, \; -10y$ are unlike terms.

Identify the like terms and unlike terms:

(i) $13x$ and $5x$

(ii) $-7m$ and $-3n$

(iii) $4x^2z$ and $-10zx^2$

(iv) $36mn$ and $-5nm$

(v) $-8p^2q$ and $3pq^2$

Activity

To identify the variables, constants, like terms and unlike terms

Make a few alphabetical cards $x, y, z, ...$ numerical cards 0, 1, 2, 3, ... and cards containing operations $+$, $-$, $\times$, $\div$ out of a chart paper and put it in a box. Call each student and ask him to do the following activity.

- Pick out the variables
- Pick out the constants
- Pick out the like terms
- Pick out the unlike terms

(v) Degree of an Algebraic expression

Consider the expression $8x^2 - 6x + 7$. It has 3 terms $8x^2, \; -6x$ and 7.

In the term $8x^2$, the power of the variable x is 2.

In the term $-6x$, the power of the variable x is 1.

The term 7 is called a constant term or an independent term.

The term 7 is $7 \times 1 = 7x^0$ in which the power of the variable x is 0.

In the above expression the term $8x^2$ has the highest power 2. So, the degree of the expression $8x^2 - 6x + 7$ is 2.

Consider the expression $6x^2y + 2xy + 3y^2$.

In the term $6x^2y$, the power of variable is 3.

(Adding the powers of x and y we get 3 (*i.e.*) 2 + 1 = 3).

In term $2xy$, the power of the variable is 2.

In term $3y^2$, the power of the variable is 2.

So, in the expression $6x^2y + 2xy + 3y^2$, the term $6x^2y$ has the highest power 3. So the degree of this expression is 3.

Hence, the degree of an expression of one variable is the highest value of the exponent of the variable. The degree of an expression of more than one variable is the highest value of the sum of the exponents of the variables in different terms.

Note: The degree of a constant is 0.

Example 2.7

The degree of the expression: (i) $5a^2 - 6a + 10$ is 2

(ii) $3x^2 + 7 + 6xy^2$ is 3

(iii) $m^2n^2 + 3mn + 8$ is 4

(vi) Value of an Algebraic expression

We know that an algebraic expression has variables and a variable can take any value. Thus, when each variable takes a value, the expression gives some value.

For example, if the cost of a book is ₹ x and if you are buying 5 books, you should pay ₹ $5x$. The value of this algebraic expression $5x$ depends upon the value of x which can take any value.

If $x = 4$, then $5x = 5 \times 4 = 20$.

If $x = 30$, then $5x = 5 \times 30 = 150$.

So to find the value of an expression, we substitute the given value of x in the expression.

Example 2.8

Find the value of the following expressions when $x = 2$.

(i) $x + 5$ (ii) $7x - 3$ (iii) $20 - 5x^2$

Solution : Substituting $x = 2$ in

(i) $x + 5 = 2 + 5 = 7$

(ii) $7x - 3 = 7(2) - 3$

$= 14 - 3 = 11$

(iii) $20 - 5x^2 = 20 - 5(2)^2$

$= 20 - 5(4)$

$= 20 - 20 = 0$

MATHEMATICS

Example 2.9

Find the value of the following expression when $a = -3$ and $b = 2$.

(i) $a + b$ (ii) $9a - 5b$ (iii) $a^2 + 2ab + b^2$

Solution Substituting $a = -3$ and $b = 2$ in

(i) $a + b = -3 + 2 = -1$

(ii) $9a - 5b = 9(-3) - 5(2)$

$= -27 - 10 = -37$

(iii) $a^2 + 2ab + b^2 = (-3)^2 + 2(-3)(2) + 22$

$= 9 - 12 + 4 = 1$

Try these

1. Find the value of the following expressions when $p = -3$

   (i) $6p - 3$ (ii) $2p^2 - 3p + 2$

2. Evaluate the expression for the given values

x	3	5	6	10
$x - 3$				

3. Find the values for the variable

x				
$2x$	6	14	28	42

Exercise 2.2

1. Choose the correct answer

   (i) The degree of the expression $5m^2 + 25mn + 4n^2$ is

   (A) 1 (B) 2 (C) 3 (D) 4

   (ii) If $p = 40$ and $q = 20$, then the value of the expression $(p - q) + 8$ is

   (A) 60 (B) 20 (C) 68 (D) 28

   (iii) The degree of the expression $x^2y + x^2y^2 + y$ is

   (A) 1 (B) 2 (C) 3 (D) 4

   (iv) If $m = -4$, then the value of the expression $3m + 4$ is

   (A) 16 (B) 8 (C) -12 (D) -8

(v) If $p = 2$ and $q = 3$, then the value of the expression $(p+q)-(p-q)$ is

(A) 6 (B) 5 (C) 4 (D) 3

2. Identify the like terms in each of the following:

   (i) $4x, 6y, 7x$

   (ii) $2a, 7b, -3b$

   (iii) $xy, 3x^2y, -3y^2, -8yx^2$

   (iv) $ab, a^2b, a^2b^2, 7a^2b$

   (v) $5pq, -4p, 3q, p^2q^2, 10p, -4p^2, 25pq, 70q, 14p^2q^2$

3. State the degree in each of the following expression:

   (i) $x^2 + yz$ (ii) $15y^2 - 3$ (iii) $6x^2y + xy$

   (iv) $a^2b^2 - 7ab$ (v) $1 - 3t + 7t^2$

4. If $x = -1$, evaluate the following:

   (i) $3x - 7$ (ii) $-x + 9$ (iii) $3x^2 - x + 7$

5. If $a = 5$ and $b = -3$, evaluate the following:

   (i) $3a - 2b$ (ii) $a^2 + b^2$ (iii) $4a^2 + 5b - 3$

2.2 Addition and subtraction of expressions

Adding and subtracting like terms

Already we have learnt about like terms and unlike terms.

The basic principle of addition is that we can add only like terms.

To find the sum of two or more like terms, we add the numerical coefficient of the like terms. Similarly, to find the difference between two like terms, we find the difference between the numerical coefficients of the like terms.

There are two methods in finding the sum or difference between the like terms namely,

(i) Horizontal method

(ii) Vertical method

(i) Horizontal method: In this method, we arrange all the terms in a horizontal line and then add or subtract by combining the like terms.

Example 2.10

Add $2x$ and $5x$.

Solution:

$2x + 5x = (2+5) \times x$

$= 7 \times x = 7x$

Group Activity

Divide the entire class into 5 groups. Ask the students of the each group to take out the things from their pencil boxes and seggregate them. Now ask them to list out the number of pens, pencils, erasers... from each box and also the total of each .

(ii) Vertical method: In this method, we should write the like terms vertically and then add or subtract.

Example 2.11

Add $4a$ and $7a$.

Solution:

$$\begin{array}{r} 4\,a \\ +\quad 7\,a \\ \hline 11\,a \\ \hline \end{array}$$

Example 2.12

Add $7pq$, $-4pq$ and $2pq$.

Solution:

Horizontal method

$7pq - 4pq + 2pq$

$= (7 - 4 + 2)\times pq$

$= 5\,pq$

Vertical method

$$\begin{array}{r} 7\,pq \\ -4\,pq \\ +2\,pq \\ \hline 5\,pq \\ \hline \end{array}$$

Example 2.13

Find the sum of $5x^2y$, $7x^2y$, $-3x^2y$, $4x^2y$.

Solution:

Horizontal method

$5x^2y + 7x^2y - 3x^2y + 4x^2y$

$= (5 + 7 - 3 + 4)x^2y$

$= 13x^2y$

Vertical method

$$\begin{array}{r} 5x^2y \\ +7x^2y \\ -3x^2y \\ +4x^2y \\ \hline 13x^2y \\ \hline \end{array}$$

Example 2.14

Subtract $3a$ from $7a$.

Solution:

Horizontal method

$7a - 3a = (7 - 3)a$

$= 4\,a$

Vertical method

$$\begin{array}{r} 7\,a \\ +3\,a \\ (-) \\ \hline 4\,a \\ \hline \end{array}$$

(Change of sign)

Do you know?

When we subtract a number from another number, we add the additive inverse to the earlier number. i.e., while subtracting 4 from 6 we change the sign of 4 to negative (additive inverse) and write as $6 - 4 = 2$.

Note: Subtracting a term is the same as adding its inverse. For example subtracting $+ 3a$ is the same as adding $- 3a$.

Example 2.15

(i) Subtract $-2xy$ from $9xy$.

Solution:

$$\begin{array}{rl} 9\,xy & \\ -\,2\,xy & \\ (+) & \text{(change of sign)} \\ \hline 11\,xy & \\ \hline \end{array}$$

(ii) Subtract

$8p^2q^2$ from $-6p^2q^2$

Solution:

$$\begin{array}{r} -6p^2q^2 \\ +\,8p^2q^2 \\ (-) \\ \hline -14p^2q^2 \\ \hline \end{array}$$

Unlike terms cannot be added or subtracted the way like terms are added or subtracted.

For example when 7 is added to x we write it as $x + 7$ in which both the terms 7 and x are retained.

Similarly, if we add the unlike terms $4xy$ and 5, the sum is $4xy + 5$. If we subtract 6 from $5pq$ the result is $5pq - 6$.

Example 2.16

Add $6a + 3$ and $4a - 2$.

Solution:

Like terms ($6a$, $4a$)

$6a + 3 + 4a - 2$

Like terms (3, -2)

$= 6a + 4a + 3 - 2$ (grouping like terms)

$= 10a + 1$

Example 2.17

Simplify : $6t + 5 + t + 1$

Solution

Like terms ($6t$, t)

$6t + 5 + t + 1$

Like terms (5, 1)

$= 6t + t + 5 + 1$ (grouping like terms)

$= 7t + 6$

Example 2.18

Add $5y + 8 + 3z$ and $4y - 5$

Solution

$5y + 8 + 3z + 4y - 5$

$= 5y + 4y + 8 - 5 + 3z$ (grouping like terms)

$= 9y + 3 + 3z$ (The term $3z$ will remain as it is.)

Example 2.19

Simplify the expression $15n^2 - 10n + 6n - 6n^2 - 3n + 5$

Solution

Grouping like terms we have

$15n^2 - 6n^2 - 10n + 6n - 3n + 5$

$= (15 - 6)n^2 + (-10 + 6 - 3)n + 5$

$= 9n^2 + (-7)n + 5$

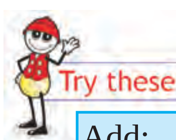
$= 9n^2 - 7n + 5$

Example 2.20

Add $10x^2 - 5xy + 2y^2$, $-4x^2 + 4xy + 5y^2$ and $3x^2 - 2xy - 6y^2$.

Solution

$$\begin{array}{r} 10x^2 - 5xy + 2y^2 \\ -4x^2 + 4xy + 5y^2 \\ +3x^2 - 2xy - 6y^2 \\ \hline 9x^2 - 3xy + y^2 \\ \hline \end{array}$$



Add:

(i) $8m - 7n$, $3n - 4m + 5$

(ii) $a + b$, $-a + b$

(iii) $4a^2$, $-5a^2$, $-3a^2$, $7a^2$

Example 2.21

Subtract $6a - 3b$ from $-8a + 9b$.

Solution

$$\begin{array}{r} -8a + 9b \\ +6a - 3b \\ (-) \quad (+) \\ \hline -14a + 12b \\ \hline \end{array}$$

Example 2.22

Subtract $2(p - q)$ from $3(5p - q + 3)$

Solution $3(5p - q + 3) - 2(p - q)$

$= 15p - 3q + 9 - 2p + 2q$

$= 15p - 2p - 3q + 2q + 9$

$= 13p - q + 9$

> **Do you know?**
>
> Just as
>
> $-(8 - 5) = -8 + 5,$
>
> $-2(m - n) = -2m + 2n$
>
> the signs of algebraic terms are handled in the same way as signs of numbers.

Example 2.23

Subtract $a^2 + b^2 - 3ab$ from $a^2 - b^2 - 3ab$.

Solution

Horizontal method

$(a^2 - b^2 - 3ab) - (a^2 + b^2 - 3ab)$

$= a^2 - b^2 - 3ab - a^2 - b^2 + 3ab$

$= -b^2 - b^2$

$= -2b^2$

Vertical method

$$\begin{array}{rrr} a^2 & - b^2 & - 3ab \\ a^2 & + b^2 & - 3ab \\ (-) & (-) & (+) \\ \hline & -2b^2 & \\ \hline \end{array}$$

Example 2.24

If $A = 5x^2 + 7x + 8$, $B = 4x^2 - 7x + 3$, find $2A - B$.

Solution $2A = 2(5x^2 + 7x + 8)$

$= 10x^2 + 14x + 16$

Now $2A - B = (10x^2 + 14x + 16) - (4x^2 - 7x + 3)$

$= 10x^2 + 14x + 16 - 4x^2 + 7x - 3$

$= 6x^2 + 21x + 13$

Subtract:

(i) $(a-b)$ from $(a+b)$

(ii) $(5x-3y)$ from $(-2x+8y)$

Example 2.25

What should be subtracted from $14b^2$ to obtain $6b^2$?

Solution

$$\begin{array}{r} 14b^2 \\ 6b^2 \\ (-)\quad \\ \hline 8b^2 \\ \hline \end{array}$$

Example 2.26

What should be subtracted from $3a^2-4b^2+5ab$ to obtain $-a^2-b^2+6ab$.

Solution

$$\begin{array}{rrr} 3a^2 & -4b^2 & +5ab \\ -a^2 & -b^2 & +6ab \\ (+) & (+) & (-) \\ \hline 4a^2 & -3b^2 & -ab \\ \hline \end{array}$$

Group Activity

Take 30 cards written with x^2, x, 1 (10 in each variety). Write on the backside of each card any one of $-x^2$, $-x$, -1.

1. Ask two students to frame 2 different expressions as told by the teacher.
2. Ask the third student to add the expressions and read out the answer.
3. Ask another student to subtract the expressions and read out the answer.

Exercise 2.3

1. Choose the correct answer :

(i) Sum of $4x$, $-8x$ and $7x$ is

(A) $5x$ (B) $4x$ (C) $3x$ (D) $19x$

(ii) Sum of 2ab, 4ab, $-8ab$ is

(A) 14ab (B) $-2ab$ (C) $2ab$ (D) $-14ab$

(iii) $5ab + bc - 3ab$ is

(A) $2ab + bc$ (B) $8ab + bc$ (C) $9ab$ (D) $3ab$

(iv) $5y - 3y^2 - 4y + y^2$ is

(A) $9y + 4y^2$ (B) $9y - 4y^2$ (C) $y + 2y^2$ (D) $y - 2y^2$

(v) If $A = 3x + 2$ and $B = 6x - 5$, then $A - B$ is

(A) $-3x + 7$ (B) $3x - 7$ (C) $7x - 3$ (D) $9x + 7$

2. Simplify :

(i) $6a - 3b + 7a + 5b$

(ii) $8l - 5l^2 - 3l + l^2$

(iii) $-z^2 + 10z^2 - 2z + 7z^2 - 14z$

(iv) $p - (p - q) - q - (q - p)$

(v) $3mn - 3m^2 + 4nm - 5n^2 - 3m^2 + 2n^2$

(vi) $(4x^2 - 5xy + 3y^2) - (3x^2 - 2xy - 4y^2)$

3. Add :

(i) $7ab,\ 8ab,\ -10ab,\ -3ab$

(ii) $s + t,\ 2s - t,\ -s + t$

(iii) $3a - 2b,\ 2p + 3q$

(iv) $2a + 5b + 7,\ 8a - 3b + 3,\ -5a - 7b - 6$

(v) $6x + 7y + 3, -8x - y - 7, 4x - 4y + 2$

(vi) $6c - c^2 + 3,\ -3c - 9,\ c^2 + 4c + 10$

(vii) $6m^2n + 4mn - 2n^2 + 5,\ n^2 - nm^2 + 3,\ mn - 3n^2 - 2m^2n - 4$

4. Subtract :

(i) $6a$ from $14a$

(ii) $-a^2b$ from $6a^2b$

(iii) $7x^2y^2$ from $-4x^2y^2$

(iv) $3xy - 4$ from $xy + 12$

(v) $m(n - 3)$ from $n(5 - m)$

(vi) $9p^2 - 5p$ from $-10p - 6p^2$

(vii) $-3m^2 + 6m + 3$ from $5m^2 - 9$

(viii) $-s^2 + 12s - 6$ from $6s - 10$

(ix) $5m^2 + 6mn - 3n^2$ from $6n^2 - 4mn - 4m^2$

5. (i) What should be added to $3x^2 + xy + 3y^2$ to obtain $4x^2 + 6xy$?

(ii) What should be subtracted from $4p + 6q + 14$ to get $-5p + 8q + 20$?

(iii) If $A = 8x - 3y + 9$, $B = -y - 9$ and $C = 4x - y - 9$ find $A + B - C$.

6. Three sides of a triangle are $3a + 4b - 2$, $a - 7$ and $2a - 4b + 3$. What is its perimeter?

7. The sides of a rectangle are $3x + 2$ and $5x + 4$. Find its perimeter.
8. Ram spends ₹$4a+3$ for a shirt and ₹$8a - 5$ for a book. How much does he spend in all?
9. A wire is $10x - 3$ metres long. A length of $3x + 5$ metres is cut out of it for use. How much wire is left out?
10. If $A = p^2 + 3p + 5$ and $B = 2p^2 - 5p - 7$, then find

    (i) $2A + 3B$ (ii) $A - B$
11. Find the value of $P - Q + 8$ if $P = m^2 + 8m$ and $Q = -m^2 + 3m - 2$.

Points to Remember

1. Algebra is a branch of Mathematics that involves alphabet, numbers and mathematical operations.
2. A variable or a literal is a quantity which can take various numerical values.
3. A quantity which has a fixed numerical value is a constant.
4. An algebraic expression is a combination of variables and constants connected by the arithmetic operations.
5. Expressions are made up of terms.
6. Terms having the same variable or product of variables with same powers are called Like terms. Terms having different variable or product of variables with different powers are called Unlike terms.
7. The degree of an expression of one variable is the highest value of the exponent of the variable. The degree of an expression of more than one variable is the highest value of the sum of the exponents of the variables in different terms

3 GEOMETRY

Geometry is a branch of Mathematics that deals with the properties of various geometrical shapes and figures. In Greek the word "Geometry" means "Earth Measurement". Geometry deals with the shape, size, position and other geometrical properties of various objects. Geometry is useful in studying space, architecture, design and engineering.

3.1. Revision

Basic Geometrical concepts:

In earlier classes you have studied about some geometrical concepts. Let us recall them.

Point

A fine dot made with a sharp pencil may be taken as roughly representing a point. A point has a position but it has no length, breadth or thickness. It is denoted by the capital letters. In the figure A, B, C, D are points.

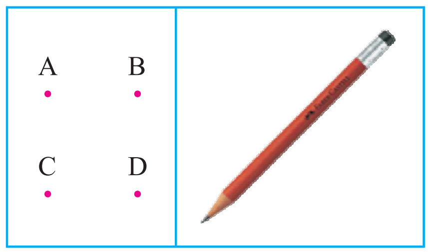


Fig. 3.1

Line

A line is traced out by a moving point. If the point of a pencil is moved over a sheet of paper, the trace left represents a line. A line has length, but it has no breadth. A line has no end points. A line AB is written as $\overleftrightarrow{AB}$. A line may be named with small letters *l*, *m*, *n*, etc. we read them as line *l*, line *m*, line *n* etc. A line has no end points as it goes on endlessly in both directions.

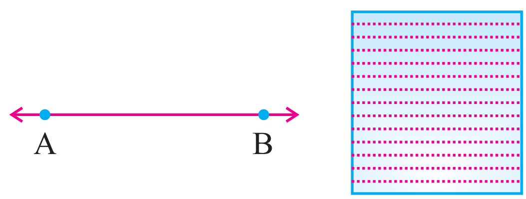


Fig. 3.2

Ray

A ray has a starting point but has no end point. The starting point is called the initial point.

Fig. 3.3

Here OA is called the ray and it is written as $\overrightarrow{OA}$. That is the ray starts from O and passes through A.

Line Segment

Let $\overleftrightarrow{AB}$ be a straight line.

Fig. 3.4

Two points C and D are taken on it. CD is a part of AB. CD is called a line segment, and is written as $\overline{CD}$. A line segment has two end points.

Plane

A plane is a flat surface which extends indefinitely in all directions. The upper surface of a table, the blackboard and the walls are some examples of planes.

3.2. Symmetry

Symmetry is an important geometrical concept commonly seen in nature and is used in every field of our life. Artists, manufacturers, designers, architects and others make use of the idea of symmetry. The beehives, flowers, tree leaves, hand kerchief, utensils have symmetrical design.

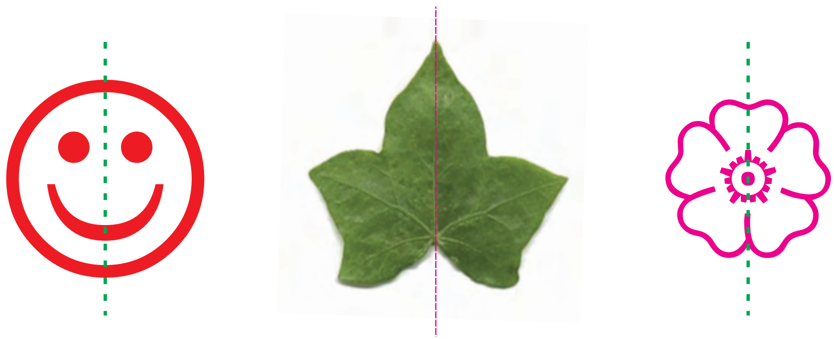


Fig. 3.5

Symmetry refers to the exact match in shape and size between two halves of an object. If we fold a picture in half and both the halves-left half and right half - match exactly then we say that the picture is symmetrical.

For example, if we cut an apple into two equal halves, we observe that two parts are in symmetry.

Fig. 3.6

Do you know?

Tajmahal in Agra is a symmetrical monument.

A butterfly is also an example of a symmetrical form. If a line is drawn down the centre of the butterfly's body, each half of the butterfly looks the same.

Fig. 3.7

Symmetry is of different types. Here we discuss about

1. Line of symmetry or axis of symmetry
2. Mirror symmetry
3. Rotational symmetry

1. Line of symmetry

In the Fig 3.8 the dotted lines divide the figure into two identical parts. If figure is folded along the line, one half of the figure will coincide exactly with the other half. This dotted line is known as line of symmetry.

When a line divides a given figure into two equal halves such that the left and right halves matches exactly then we say that the figure is symmetrical about the line. This line is called the line of symmetry or axis of symmetry.

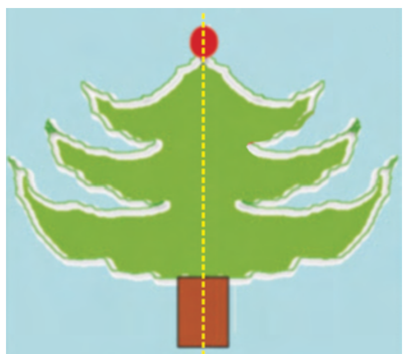


Fig. 3.8

Activity 1:

Take a rectangular sheet of paper. Fold it once lengthwise, so that one half fits exactly over the other half and crease the edges. Now open it, and again fold it once along its width.

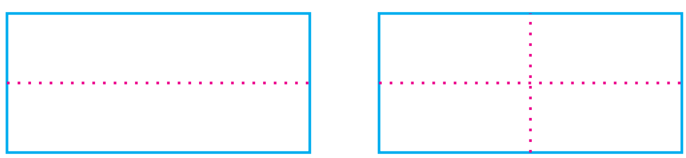


Fig. 3.9

In this paper folding,

You observe that a rectangle has two lines of symmetry.

Discuss: Does a parallelogram have a line of symmetry?

Activity 2:

One of the two set squares in your geometry box has angle of measure $30^0, 60^0, 90^0$. Take two such identical set squares. Place them side by side to form a 'kite' as shown in the Fig. 3.10.

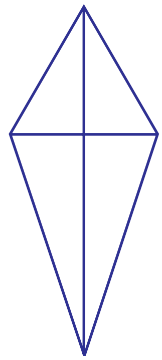


Fig. 3.10

How many lines of symmetry does the shape have?

You observe that this kite shape figure has one line of symmetry about its vertical diagonal.

Activity 3:

For the given regular polygons find the lines of symmetry by using paper folding method and also draw the lines of symmetry by dotted lines.

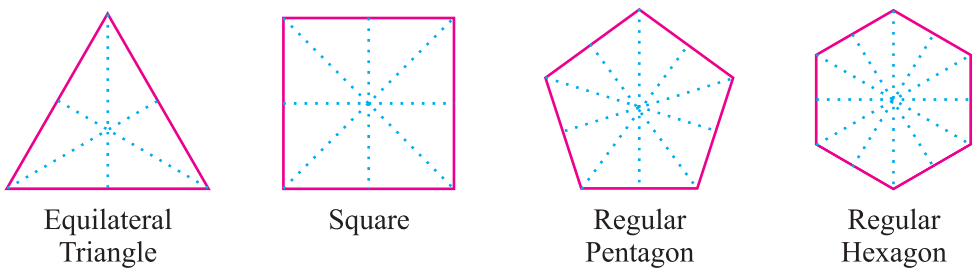


Fig. 3.11

In the above paper foldings, you observe that

(i) An equilateral triangle has three lines of symmetry.

(ii) A square has four lines of symmetry

(iii) A regular pentagon has five lines of symmetry.

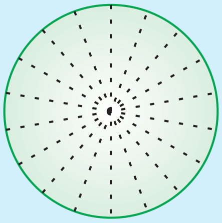
(iv) A regular hexagon has six lines of symmetry.

Do you know?

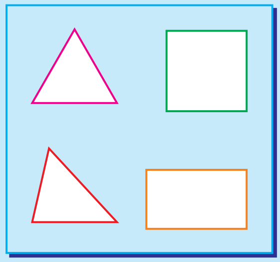
A polygon is said to be regular if all its sides are of equal length and all its angles are of equal measure.

Each regular polygon has as many lines of symmetry as it has sides.

Do you know?

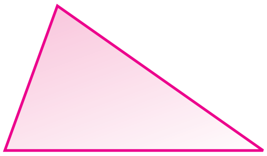


A circle has many lines of symmetry.

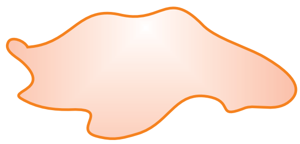


Identify the regular polygon

Some objects and figures have no line of symmetry.



Scalene triangle



Irregular shape

Fig. 3.12

Make a list of English alphabets which have no line of symmetry

The above figures have no line of symmetry; because these figures are not symmetrical. We can say that these figures are asymmetrical.

Do you know? To reflect an object means to produce its mirror image.

2. Mirror line symmetry

When we look into a mirror we see our image is behind the mirror. This image is due to reflection in the mirror. We know that the image is formed as far behind the mirror as the object is in front of it.

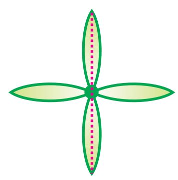


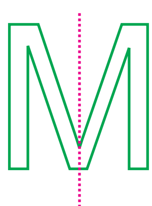
Fig. 3.13

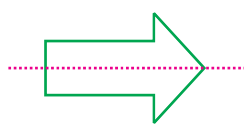
In the above figure if a mirror is placed along the line at the middle, the half part of the figure reflects through the mirror creating the remaining identical half. In other words, the line where the mirror is placed divides the figure into two identical parts in Fig. 3.13. They are of the same size and one side of the line will have its reflection exactly at the same distance on the other side. Thus it is also known as mirror line symmetry.

While dealing with mirror reflection, we notice that the left-right changes as seen in the figure.

Example 3.1

The figure shows the reflection of the mirror lines.





Exercise 3.1

1. Choose the correct answer :

i) An isosceles triangle has
   (A) no lines of symmetry (B) one line of symmetry
   (C) three lines of symmetry (D) many lines of symmetry

ii) A parallelogram has
   (A) two lines of symmetry (B) four lines of symmetry
   (C) no lines of symmetry (D) many lines of symmetry

iii) A rectangle has
   (A) two lines of symmetry (B) no lines of symmetry
   (C) four lines of symmetry (D) many lines of symmetry

iv) A rhombus has
   (A) no lines os symmetry (B) four lines of symmetry
   (C) two lines of symmetry (D) six lines of symmetry

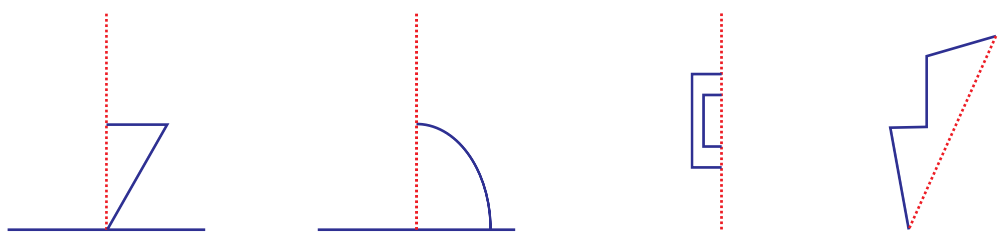
v) A scalene triangle has
   (A) no lines of symmetry (B) three lines of symmetry
   (C) one line of symmetry (D) many lines of symmetry

2. Which of the following have lines of symmetry?

(i) (ii) (iii) (iv)

How many lines of symmetry does each have?

3. In the following figures, the mirror line (i.e. the line of symmetry) is given in dotted line. Complete each figure performing reflection in the dotted (mirror) line.



(i) (ii) (iii) (iv)

4. Complete the following table:

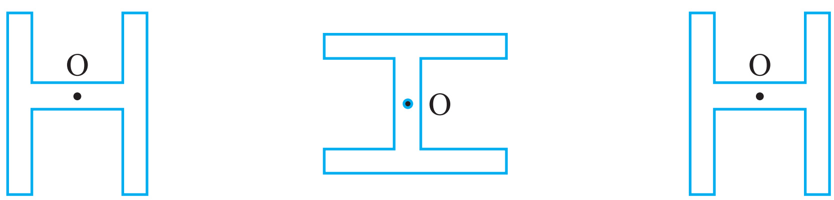
Shape	Rough figure	Number of lines of symmetry
Equilateral triangle		
Square		
Rectangle		
Isosceles triangle		
Rhombus		

5. Name a triangle which has
   (i) exactly one line of symmetry.
   (ii) exactly three lines of symmetry.
   (iii) no lines of symmetry.

6. Make a list of the capital letters of English alphabets which
   (i) have only one line of symmetry about a vertical line.
   (ii) have only one line of symmetry about a horizontal line.
   (iii) have two lines of symmetry about both horizontal and vertical line of symmetry.

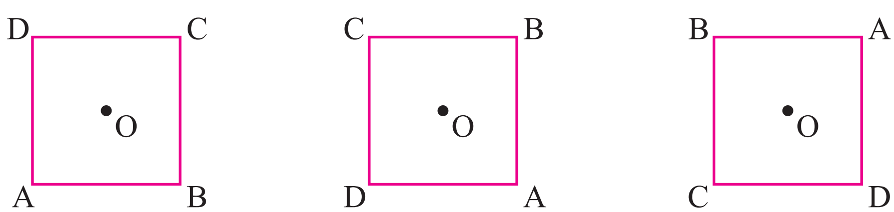
3.3 Rotational Symmetry

Look at the following figures showing the shapes that we get, when we rotate about its centre 'O' by an angle of 90^0 or 180^0



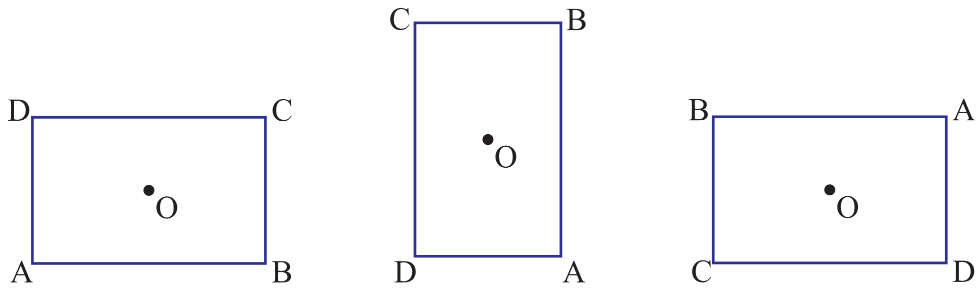
Letter H | 90° rotation | 180° rotation

Fig. 3.14



Square | 90° rotation | 180° rotation

Fig. 3.15



Rectangle 90° rotation 180° rotation

Fig. 3.16

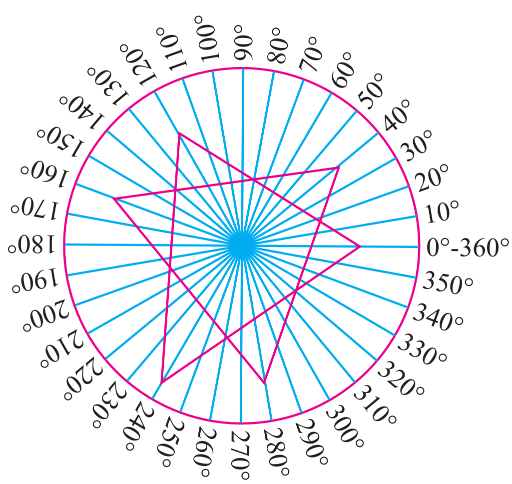
In the case of a square, we get exactly the same shape after it is rotated by 90^0 while in the case of a rectangle, we get exactly the same shape after it is rotated by 180° such figures which can be rotated through an angle less than 360° to get the same shape are said to have rotational symmetry.

Angle of Rotation

The minimum angle through which the figure has to be rotated to get the original figure is called the angle of rotation and the point about which the figure is rotated is known as centre of rotation.

Activity 4:

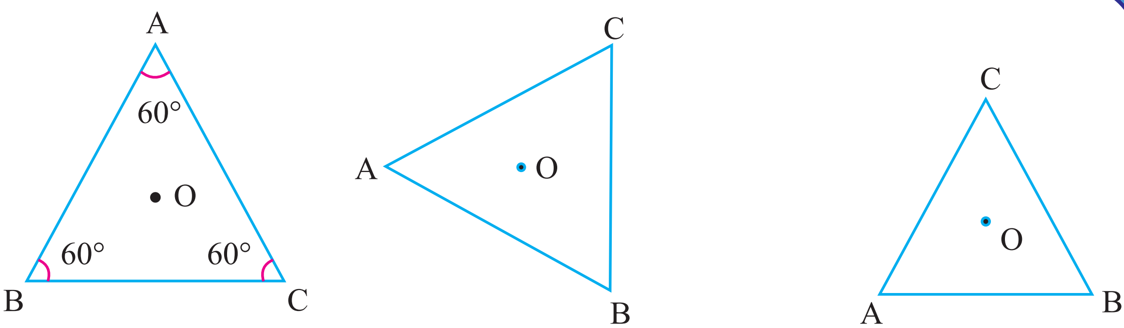
Take two card board sheets and cut off one equilateral triangle in each sheet such that both the triangles are identical. Prepare a circle on a card board and mark the degrees from 0 to 360 degree in the anticlockwise direction. Now place one triangle exactly over the other and put a pin through the centres of the figures. Rotate the top figure until it matches with the lower figure.



You observe that the triangle has been rotated through an angle 120°.

Again rotate the top figure until it matches with the lower figure for the second time. Now you observe that the top of figure has been rotated through an angle 240° from the original position.

Rotate the top figure for the third time to match with the lower figure. Now the top triangle has reached its original position after a complete rotation of 360° From the above activity you observe that an equilateral triangle has angle of rotation 120°.



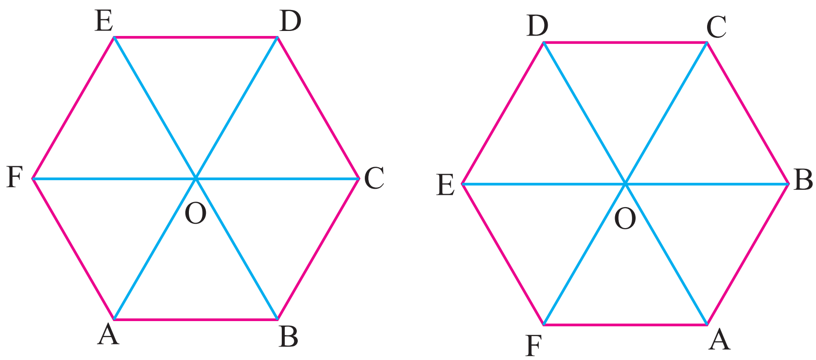
Equilateral triangle

60° rotation

120° rotation

Fig. 3.17

Angle of rotation of a hexagon



Hexagon

60° rotation

Fig. 3.18

In the above Fig. 3.15 to 3.18.

We get exactly the same shape of square, rectangle, equilateral triangle and hexagon after it is rotated by $90^0, 180^0, 120^0, 60^0$ respectively.

Thus the angle of rotation of

(i) a square is 90^0

(ii) a rectangle is 180^0

(iii) an equilateral triangle is 120^0

(iv) a hexagon is 60^0

Order of rotational symmetry

The order of rotational symmetry is the number that tell us how many times a figure looks exactly the same while it takes one complete rotation about the centre.

Thus if the angle of rotation of an object is x^0

Its order of rotational symmetry $= \frac{360}{x^0}$

In Fig. 3.15 to 3.18.

The order of rotational symmetry of

(i) a square is $\frac{360^0}{90^0} = 4$

(ii) a rectangle is $\frac{360^0}{180^0} = 2$

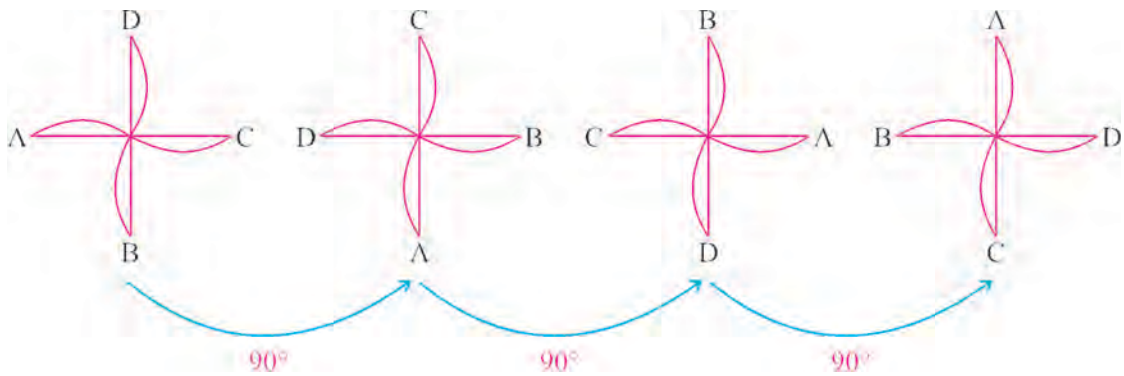
(iii) an equilateral triangle is $\frac{360^0}{120} = 3$

(iv) a hexagon is $\frac{360^0}{60^0} = 6.$

Example 3.2

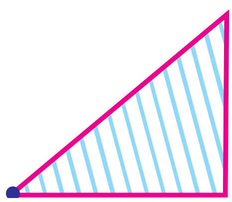
The objects having no line of symmetry can have rotational symmetry.

Have you ever made a paper wind mill? The paper wind mill in the picture looks symmetrical. But you do not find any line of symmetry. No folding can help you to have coincident halves. However if you rotate it by 90° about the the centre, the windmill will look exactly the same. We say the wind mill has a rotational symmetry.



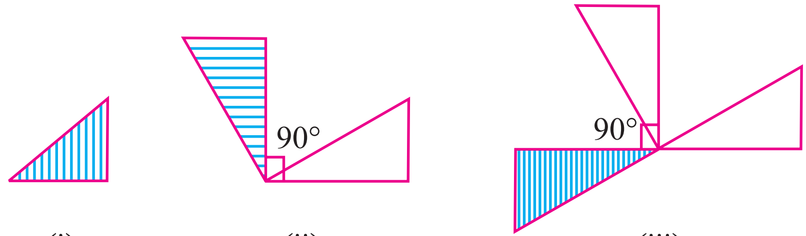
In a full turn, there are four positions (on rotation through the angles $90^0, 180^0 270^0$ and 360^0) in which the wind mill looks exactly the same. Because of this, we say it has a rotational symmetry of order 4.

Activity 5:

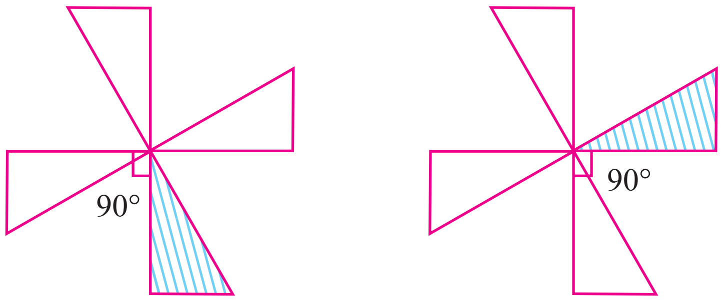


As shown in figure cut out a card board or paper triangle. Place it on a board and fix it with a drawing pin at one of its vertices. Now rotate the triangle about this vertex, by 90^0 at a time till it comes to its original position.

You observe that, for every 90^0 you have the following figures (ii to v).



(i) (ii) (iii)



(iv) (v)

The triangle comes back to its original position at position (v) after rotating through 360^0. Thus the angle of rotation of this triangle is 360^0 and the order of rotational symmetry of this triangle is $\frac{360^0}{360^0} = 1$.

Exercise 3.2

1. Choose the correct answer:

i) The angle of rotation of an equilateral triangle is

(A) 60^0 (B) 90^0 (C) 120^0 (D) 180^0

ii) The order of rotational symmetry of a square is

(A) 2 (B) 4 (C) 6 (D) 1.

iii) The angle of rotation of an object is 72^0 then its order of rotational symmetry is

(A) 1 (B) 3 (C) 4 (D) 5

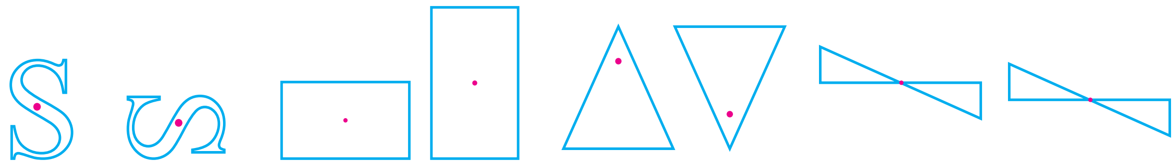
iv) The angle of rotation of the letter 'S' is

(A) 90^0 (B) 180^0 (C) 270^0 (D) 360^0

v) the order of rotational symmetry of the letter 'V' is one then its angle of rotation is

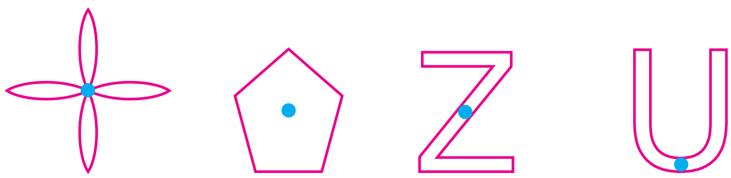
(A) 60^0 (B) 90^0 (C) 180^0 (D) 360^0

2. The following figures make a rotation to come to the new position about a given centre of rotation. Examine the angle through which the figure is rotated.



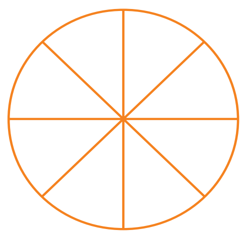
(i) (ii) (iii) (iv)

3. Find the angle of rotation and the order of rotational symmetry for the following figures given that the centre of rotation is 'O'.



(i) (ii) (iii) (iv)

4. A circular wheel has eight spokes.



What is the angle of rotation and the order of rotation?

3.3 Angle

Two rays starting from a common point form an angle. In $\angle AOB$, O is the vertex, $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the two rays.

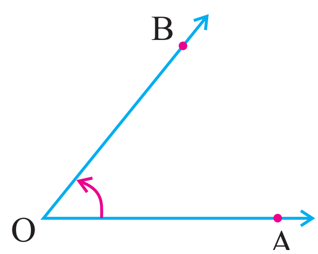


Fig. 3.19

Types of angles

(i) Acute angle:

An angle whose measure is greater than 0° but less than 90^{0} is called an acute angle.

Example: $15^{0}, 30^{0}, 60^{0}, 75^{0}$, In Fig. 3.20 $\angle AOB = 30^{0}$ is an acute angle.

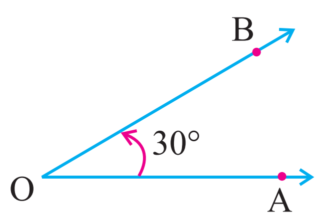


Fig. 3.20

(ii) Right angle

An angle whose measure is 90^0 is called a right angle.

In Fig. 3.21 $\angle AOB = 90^0$ is a right angle.

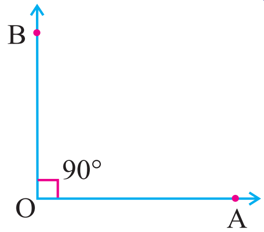


Fig. 3.21

(iii) Obtuse angle

An angle whose measure is greater than 90^0 and less than 180^0 is called an obtuse angle.

Example: $100^0, 110^0, 120^0, 140^0$

In Fig. 3.22 $\angle$ AOB = 110^0 is an obtuse angle.

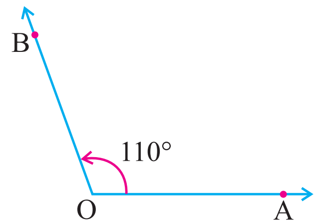


Fig. 3.22

(iv) Straight angle

When the rays of an angle, are opposite rays forming a straight line. The angle thus formed is a straight angle and whose measure is 180^0 In Fig. (3.23) $\angle$ AOB = 180^0 is a straight angle.

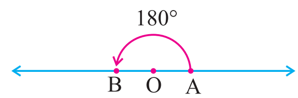


Fig. 3.23

(v) Reflex angle

An angle whose measure is more than 180^0 but less than 360^0 is called a reflex angle. In Fig. 3.24 $\angle AOB = 220°$ is a reflex angle.

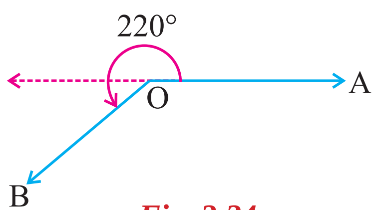


Fig. 3.24

(vi) Complete angle

In Fig. 3.25

The angle formed by $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ is one complete circle, that is 360^0.Such an angle is called a complete angle

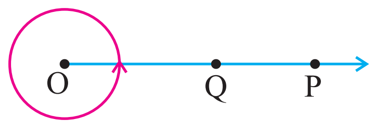


Fig. 3.25

Related Angles

(i) Complementary angles

If the sum of the measures of two angle is 90^0, then the two angles are called complementary angles. Here each angle is the complement of the other.

The complement of 30^0 is $60^0$and the complement of 60^0 *is* 30^0

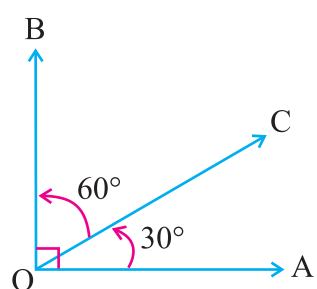


Fig. 3.26

(ii) Supplementary angles

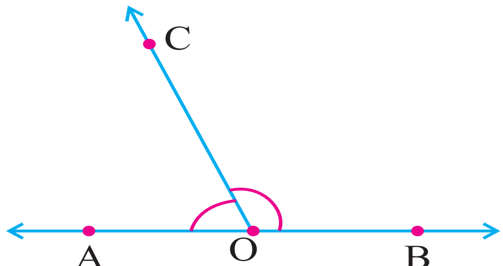
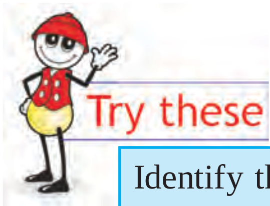


Fig. 3.27

If the sum of the measures of two angle is 180^0, then the two angles are called supplementary angles. Here each angle is the supplementary of the other.

The supplementary angle of 120^0 is 60^0 and the supplementary angle of 60° is 120^0



Identify the following pairs of angles are complementary or supplementary

(a) 80^0 and 10^0 _____

(b) 70^0 and 110^0 _____

(c) 40^0 and 50^0 ______

(d) 95^0 and 85^0 _______

(e) 65^0 and 115^0 ______

Fill in the blanks.

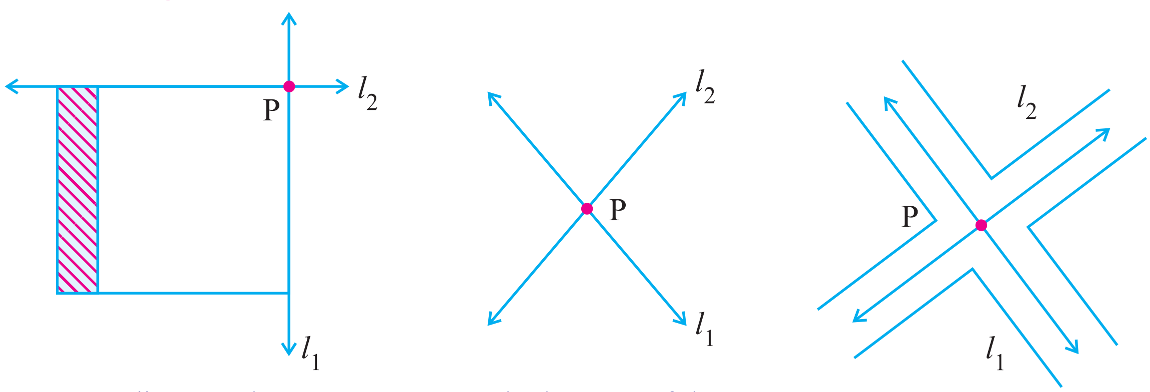
(a) Complement of 85^0 *is*____

(b) Complement of 30° is ___

(c) Supplement of 60^0 is ____

(d) Supplement of 90^0 *is*_____

Intersecting lines



Two adjacent edges of your book

The letter X of the English alphabet

Crossing Roads

Fig. 3.28

Look at the Fig. 3.28. Two lines l_1 and l_2 are shown. Both the lines pass through a point P. We say l_1 and l_2 intersect at P. If two lines have one common point, they are called intersecting lines. The common point 'P' is their point of intersection.

Angles in intersecting lines

When two lines intersect at a point angles are formed.

In Fig. 3.29 the two lines AB and CD intersect at a point 'O', ∠COA, ∠AOD, ∠DOB, ∠BOC are formed. Among the four angles two angles are acute and the other two angles are obtuse. But in figure 3.30 if the two intersecting lines are perpendicular to each other then the four angles are at right angles.

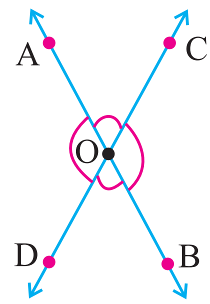


Fig. 3.29

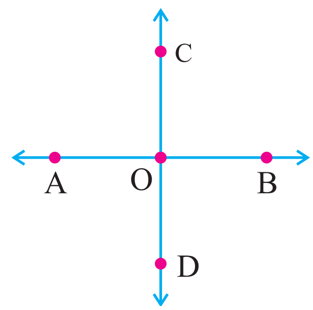


Fig. 3.30

Adjacent angles

If two angles have the same vertex and a common ray, then the angles are called adjacent angles.

In Fig. 3.31 ∠BAC and ∠CAD are adjacent angles (i.e . $\angle x$ and $\angle y$) as they have a common ray $\overrightarrow{AC}$, a common vertex A and both the angle ∠BAC and ∠CAD are on either side of the common ray $\overrightarrow{AC}$.

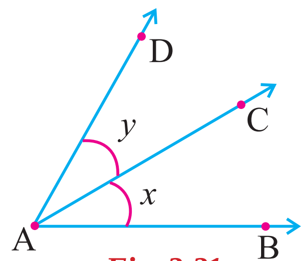
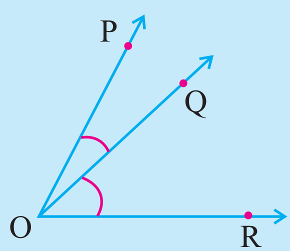
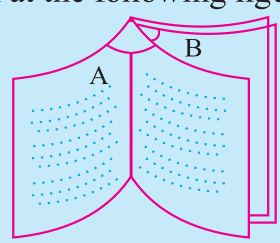


Fig. 3.31

∠ ROP and ∠QOP are not adjacent angle. Why?



Look at the following figure



Open a book looks like the above figure. Is the pair of angles are adjacent angles?

(i) Adjacent angles on a line.

When a ray stands on a straight line two angles are formed. They are called linear adjacent angles on the line.

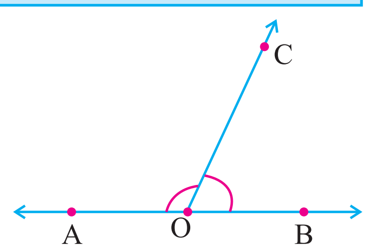


Fig. 3.32

In Fig. 3.32 the ray OC stands on the line AB. ∠BOC and ∠COA are the two adjacent angles formed on the line AB. Here 'O' is called the common vertex, $\overrightarrow{OC}$ is called the common arm. The rays OA and OB lie on the opposite sides of the common ray OC.

Two angles are said to be linear adjacent angles on a line if they have a common vertex, a common ray and the other two rays are on the opposite sides of the common ray.

(ii) The sum of the adjacent angles on a line is 180°

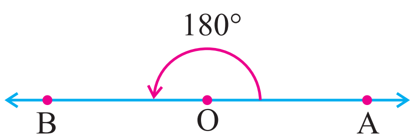


Fig. 3.33

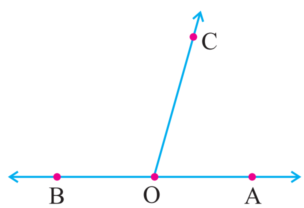


Fig.3.34

In Fig. 3.33 ∠AOB = 180^0 is a straight angle.

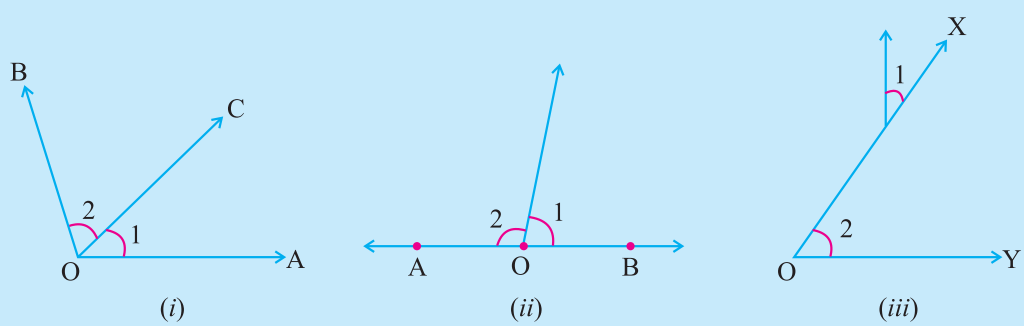
In Fig. 3.34 The ray OC stands on the line AB. ∠AOC and ∠COB are adjacent angles. Since ∠ AOB is a straight angle whose measure is 180^0

∠AOC + ∠ COB = 180^0

From this we conclude that the sum of the adjacent angles on a line is 180^0

Note 1: A pair of adjacent angles whose non common rays are opposite rays.

Note 2: Two adjacent supplementary angles form a straight angle.



Are the angles marked 1 and 2 adjacent?

If they are not adjacent, Justify your answer.

Do you know?

A vegetable chopping board

A pen stand

The chopping blade makes a linear pair of angles with the board.

The pen makes a linear pair of angles with the stand.

(i) Can two adjacent acute angles form a liner pair?

(ii) Can two adjacent obtuse angles form a linear pair?

(iii) Can two adjacent right angles form a linear pair?

(iv) Can an acute and obtuse adjacent angles form a linear pair?

(iii) Angle at a point

In Fig. 3.35, four angles are formed at the point 'O'. The sum of the four angles formed is 360^0.

(i.e) $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^0$

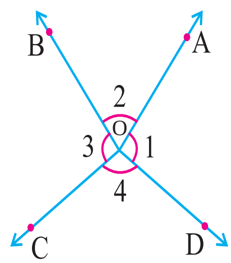


Fig. 3.35

(iv) Vertically opposite angles

If two straight lines AB and CD intersect at a point 'O'. Then $\angle$AOC and $\angle$BOD form one pair of vertically opposite angles and $\angle$ DOA and $\angle$COB form another pair of vertically opposite angles.

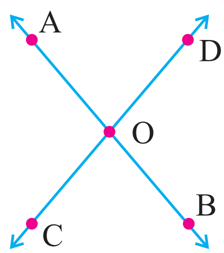


Fig. 3.36

Do you know?

The following are some real life example for vertically Opposite angles

Activity 6: Draw two lines '*l*' and '*m*', intersecting at a point 'P' mark $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$ as in the Fig. 3.37.

Take a trace copy of the figure on a transparent sheet. Place the copy on the original such that $\angle$ 1 matches with its copy, $\angle 2$, matches with its copy.. etc...

Fix a pin at the point of intersection of two lines '*l*' and '*m*' at P. Rotate the copy by 180^0. Do the lines coincide again?

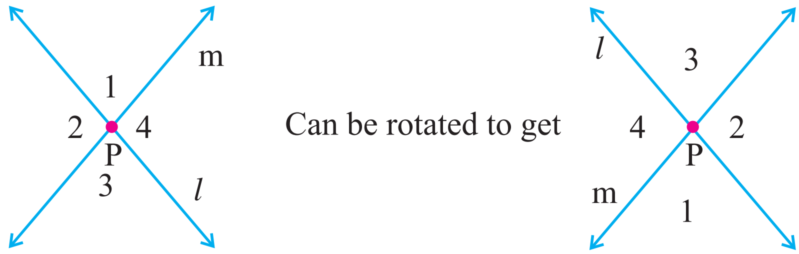


Fig. 3.37

You find that $\angle 1$ and $\angle 3$ have interchanged their positions and so have $\angle 2$ and $\angle 4$. (This has been done without disturbing the position of the lines).

Thus $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

From this we conclude that when two lines intersect, the vertically opposite angles are equal.

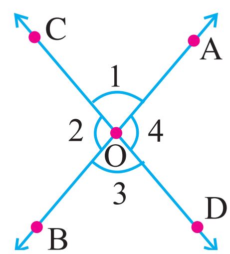


Fig. 3.38

Now let us try to prove this using Geometrical idea.

Let the lines AB and CD intersect at 'O' making angles $\angle 1$, $\angle 2$, $\angle 3$ and $\angle 4$.

Now $\angle 1 = 180^0 - \angle 2 \rightarrow$ (i)

(Since sum of the adjacent angle on a line 180^0)

$\angle 3 = 180^0 - \angle 2 \rightarrow$ (ii)

(Since sum of the adjacent angle on a line 180^0).

From (i) and (ii)

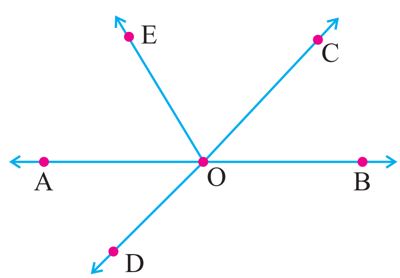
$\angle 1 = \angle 3$ and similarly we prove that $\angle 2 = \angle 4$.

Example 3.3

In the given figure identify

(a) Two pairs of adjacent angles.

(b) Two pairs of vertically opposite angles.



Solution

(a) Two pairs of adjacent angles are

(i) $\angle EOA$, $\angle COE$ since OE is common to $\angle$ EOA and $\angle COE$

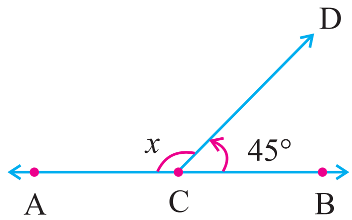
(ii) $\angle COA$, $\angle BOC$ since OC is common to $\angle COA$ and $\angle BOC$

(b) Two pairs of vertically opposite angles are i) $\angle BOC$, $\angle AOD$

ii) $\angle COA$, $\angle DOB$.

Example 3.4

Find the value of x in the given figure.



Solution

$\angle BCD + \angle DCA = 180^0$

(Since $\angle BCA = 180^0$ is a straight angle)

$45° + x = 180°$

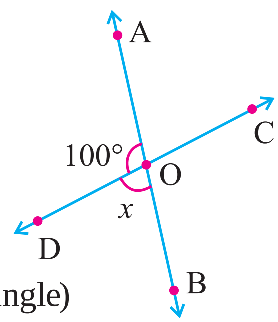
$x = 180° - 45°$

$= 135°$

$\therefore$ The value of x is 135^0.

Example 3.5

Find the value of x in the given figure.



Solution

$\angle AOD + \angle DOB = 180^0$

(Since $\angle AOB = 180^0$ is a straight angle)

$100^0 + x = 180^0$

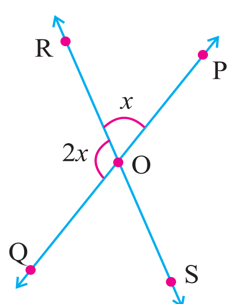
$x = 180^0 - 100^0$

$= 80^0$

$\therefore$ The value of x is 80^0.

Example 3.6

Find the value of x in the given figure.



Solution

$\angle POR + \angle ROQ = 180^0$

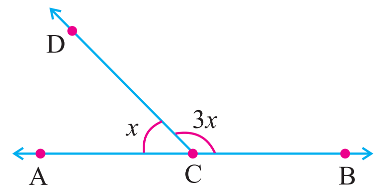
(Since $\angle POQ = 180^0$ is a straight angle)

$$x + 2x = 180^0$$
$$3x = 180^0$$
$$x = \frac{180^0}{3}$$
$$= 60^0$$

$\therefore$ The value of x is 60^0

Example 3.7

Find the value of x in the given figure.



Solution

$\angle BCD + \angle DCA = 180^0$

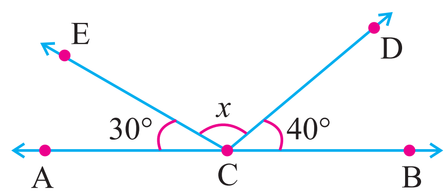
(Since $\angle BCA = 180^0$ is a straight angle)

$$3x + x = 180^0$$
$$4x = 180^0$$
$$x = \frac{180^0}{4}$$
$$= 45^0$$

$\therefore$ The value of x is 45^0

Example 3.8

Find the value of x in the given figure.



Solution

$\angle BCD + \angle DCE + \angle ECA = 180^0$

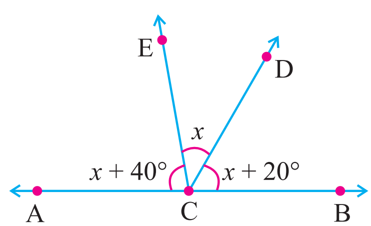
(Since $\angle BCA = 180^0$ is a straight angle)

$$40^0 + x + 30^0 = 180^0$$
$$x + 70^0 = 180^0$$
$$x = 180^0 - 70^0$$
$$= 110^0$$

$\therefore$ The value of x is 110^0

Example 3.9

Find the value of x in the given figure.



Solution

$\angle BCD + \angle DCE + \angle ECA = 180^0$ (Since $\angle BCA = 180^0$ straight angle).

$$x + 20^0 + x + x + 40^0 = 180^0$$

$$3x + 60^0 = 180^0$$

$$3x = 180^0 - 60^0$$

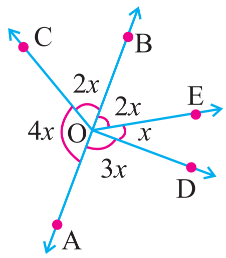
$$3x = 120^0$$

$$x = \frac{120}{3} = 40^0$$

$\therefore$ The value of x is 40^0

Example 3.10

Find the value of x in the given figure.



Solution

$$\angle BOC + \angle COA + \angle AOD + \angle DOE + \angle EOB = 360^0$$

(Since angle at a point is 360^0)

$$2x + 4x + 3x + x + 2x = 360^0$$

$$12x = 360^0$$

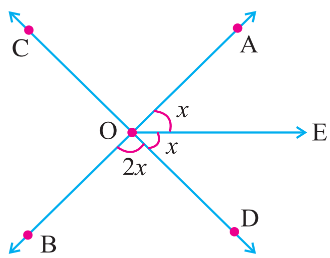
$$x = \frac{360^0}{12}$$

$$= 30^0$$

$\therefore$ The value of x is 30^0

Example 3.11

Find the value of x in the given figure.



Solution

$$\angle BOD + \angle DOE + \angle EOA = 180^0$$

(Since $\angle$ AOB = 180^0 is straight angle)

$$2x + x + x = 180^0$$

$$4x = 180^0$$

$$x = \frac{180^0}{4}$$

$$= 45^0$$

$\therefore$ The value of x is 45^0

Exercise: 3.3

1. Choose the correct answer:

i) The number of points common to two intersecting line is

(A) one (B) Two (C) three (D) four

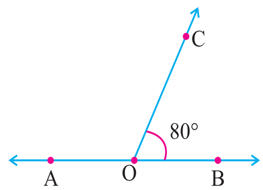
ii) The sum of the adjacent angles on a line is

(A) 90^0 (B) 180^0 (C) 270^0 (D) 360^0

iii) In the figure $\angle COA$ will be

(A) 80^0 (B) 90^0

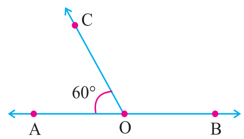
(C) 100^0 (D) 95^0



iv) In the figure $\angle BOC$ will be

(A) 80^0 (B) 90^0

(C) 100^0 (D) 120^0

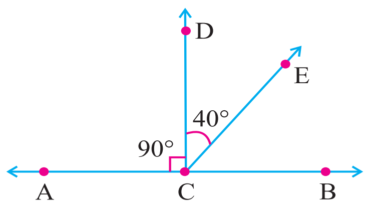


v) In the figure CD is perpendicular to AB.

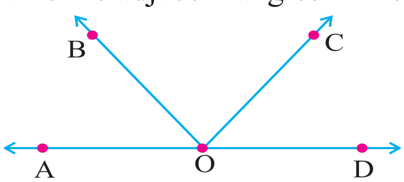
Then the value of $\angle BCE$ will be

(A) 45^0 (B) 35^0

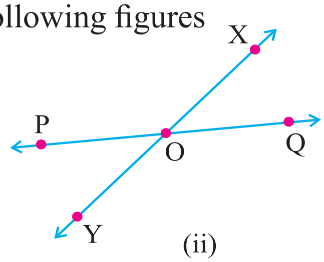
(C) 40^0 (D) 50^0



2. Name the adjacent angles in the following figures

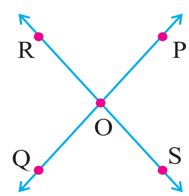


(i)



(ii)

3. Identify the vertically opposite angles in the figure:



4. Find $\angle B$ if $\angle A$ measures?

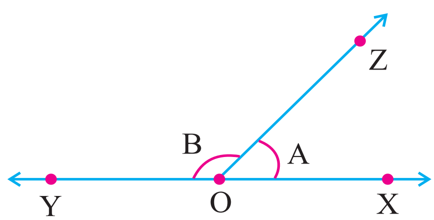
(i) 30^0

(ii) 80^0

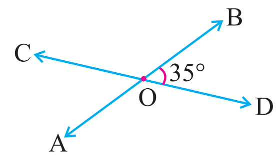
(iii) 70°

(iv) 60°

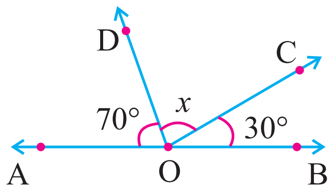
(v) 45^0



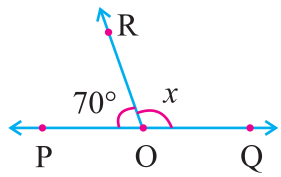
5. In figure AB and CD be the intersecting lines if $\angle DOB = 35^{0}$ find the measure of the other angles.



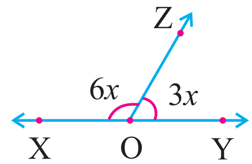
6. Find the value of x in the following figures :



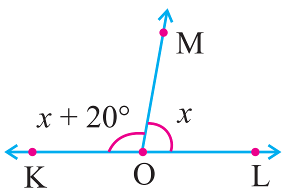
(i)



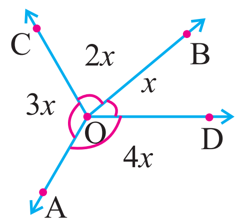
(ii)



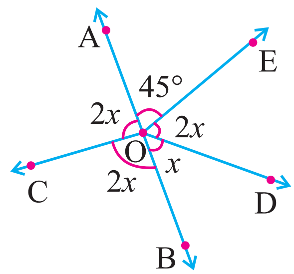
(iii)



(iv)

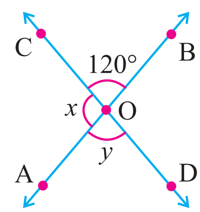


(v)



(vi)

7. In the following figure two lines AB and CD intersect at the point O. Find the value of x and y.



8. Two linear adjacent angles on a line are $4x$ and $(3x + 5)$. Find the value of x.

Points to Remember

1. Symmetry refers to the exact match in shape and size between two halves of an object.
2. When a line divides a given figure into two equal halves such that the left and right halves matches exactly then we say that the figure is symmetrical about the line. This line is called the line of symmetry or axis of symmetry.
3. Each regular polygon has as many lines of symmetry as it has sides.
4. Some objects and figures have no lines of symmetry.
5. Figures which can be rotated through an angle less than 360° to get the same shape are said to have rotational symmetry.
6. The order of rotational symmetry is the number that tell us how many times a figure looks exactly the same while it takes one complete rotation about the centre.
7. The objects having no line of symmetry can have rotational symmetry.
8. If two angles have the same vertex and a common ray, then the angles are called adjacent angles.
9. The sum of the adjacent angles on a line is 180°.
10. When two lines intersect, the vertically opposite angles are equal.
11. Angle at a point is 360°.

4 PRACTICAL GEOMETRY

4.1 Introduction

This chapter helps the students to understand and confirm the concepts they have learnt already in theoretical geometry. This also helps them to acquire some basic knowledge in geometry which they are going to prove in their later classes. No doubt, all the students will do the constructions actively and learn the concepts easily.

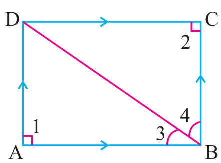
In the previous class we have learnt to draw a line segment, the parallel lines, the perpendicular lines and also how to construct an angle.

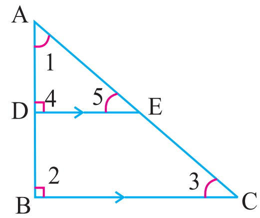
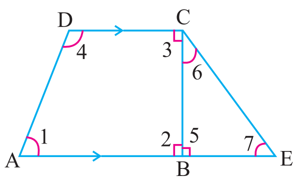
Here we are going to learn about the construction of perpendicular bisector of a line segment, angle bisector, some angles using scale and compass and the construction of triangles.

Review

To recall the concept of angles, parallel lines and perpendicular lines from the given figure.

We shall identify the points, the line segments, the angles, the parallel lines and the perpendicular lines from the figures given below in the table.

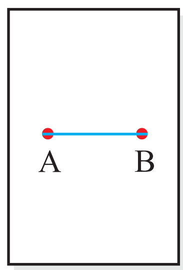
S. No.	Figures	Points identi-fied	Line segment identified	Angles identified	Parallel lines	Perpendic-ular lines
1		A, B, C and D	AB, BC, CD, AD, and BD	1 - ∠BAD (∠A) 2 - ∠DCB (∠C) 3 - ∠DBA 4 - ∠CBD	AB ∥ DC BC ∥ AD	AB ⊥ AD AB ⊥ BC BC ⊥ CD CD ⊥ AD

Sl. No.	Figures	Points iden-tified	Line segment identified	Angles identified	Parallel lines	Perpen-dicular lines
1.						
2.						

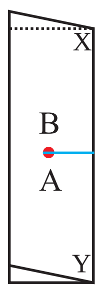
4.2 Perpendicular bisector of the given line segment

(i) Activity : Paper folding

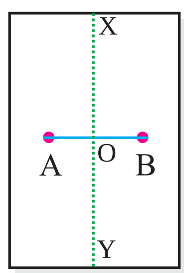
- Draw a line segment AB on a sheet of paper.



- Fold the paper so that the end point B lies on A. Make a crease XY on the paper.



- Unfold the paper. Mark the point O where the line of crease XY intersects the line AB.



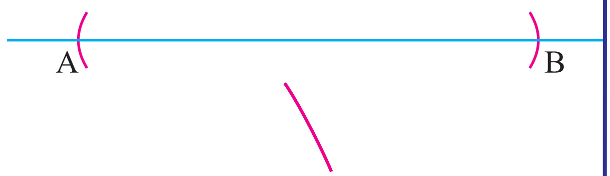
- By actual measurement we can see that OA = OB and the line of crease XY is perpendicular to the line AB.

The line of crease XY is the perpendicular bisector of the line AB.

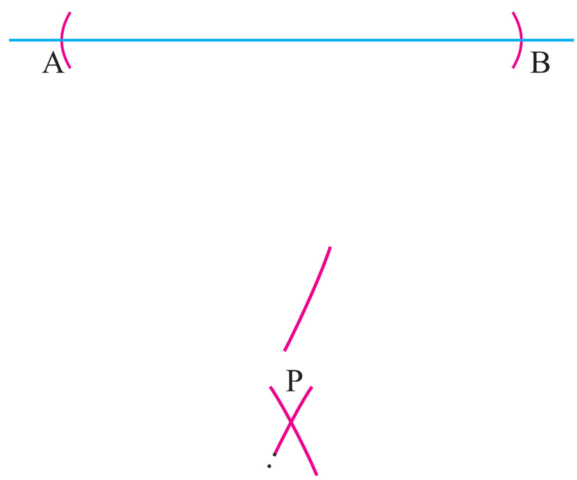
> The perpendicular bisector of a line segment is a perpendicular line drawn at its midpoint.

(ii) To construct a perpendicular bisector to a given line segment.

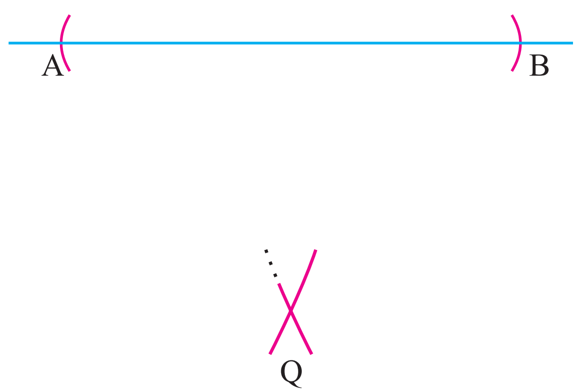
Step 1 : Draw a line segment AB of the given measurement.



Step 2 : With 'A' as centre draw arcs of radius more than half of AB, above and below the line AB.

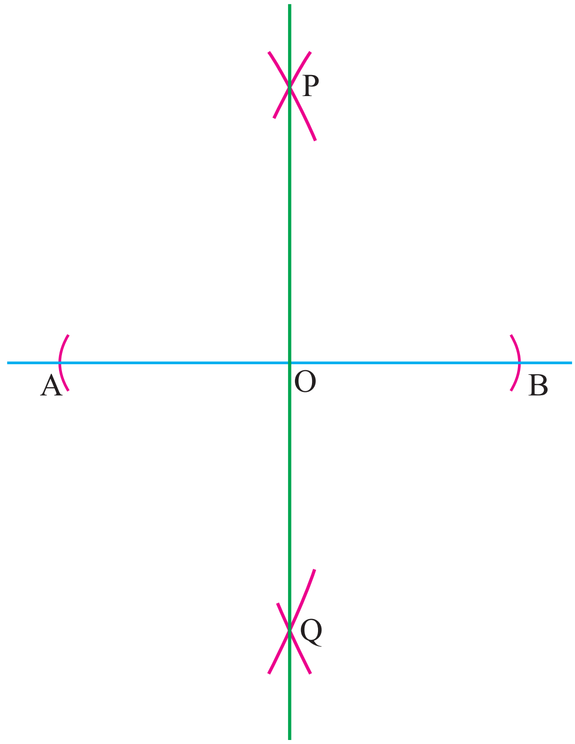


Step 3 : With 'B' as centre and with the same radius draw two arcs. These arcs cut the previous arcs at P and Q.



Step 4 : Join PQ. Let PQ intersect AB at ‘O’.

PQ is a perpendicular bisector of AB.



Try these

Mark any point on the perpendicular bisector PQ. Verify that it is equidistant from both A and B.

Do you know?

The perpendicular bisector of a line segment is the axis of symmetry for the line segment.

Think !

Can there be more than one perpendicular bisector for the given line segment?

Example 4.1

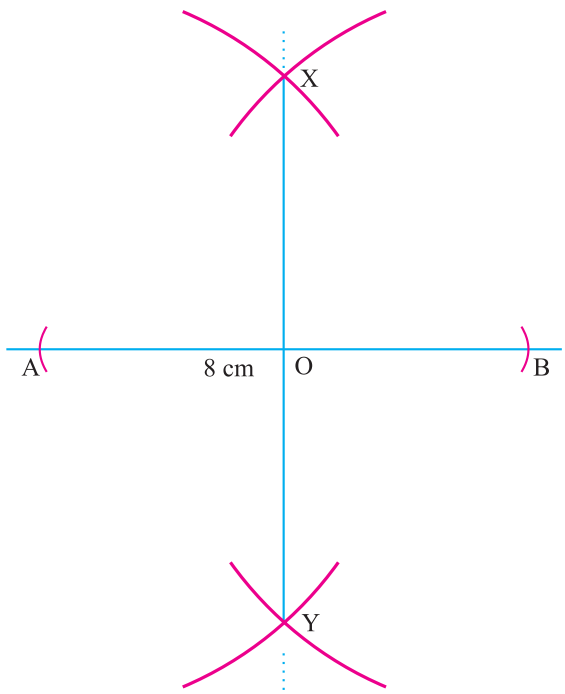
Draw a perpendicular bisector to the line segment AB = 8 cm.

Solution

Step 1 : Draw the line segment AB = 8cm.

Step 2 : With ‘A’ as centre draw arcs of radius more than half of AB above and below the line AB.

Step 3 : With ‘B’ as centre draw the arcs of same radius to cut the previous arcs at X and Y.



Step 4 : Join XY to intersect the line AB at O.

XY is the perpendicular bisector of AB.

1. With PQ = 6.5 cm as diameter draw a circle.
2. Draw a line segment of length 12 cm. Using compass divide it into four equal parts. Verify it by actual measurement.
3. Draw a perpendicular bisector to a given line segment AC. Let the bisector intersect the line at 'O'. Mark the points B and D on the bisector at equal distances from O. Join the points A, B, C and D in order. Verify whether all lines joined are of equal length.

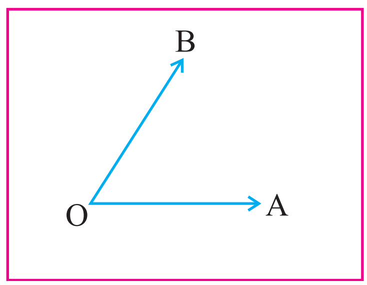
Think!

In the above construction mark the points B and D on the bisector, such that OA = OB = OC = OD. Join the points A, B, C and D in order. Then

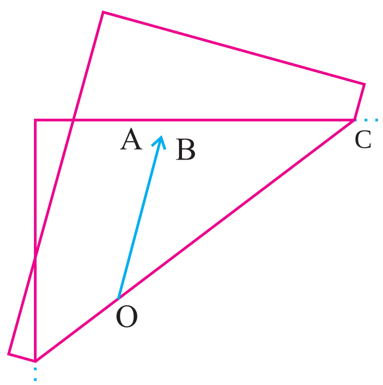
1. Do the lines joined are of equal length?
2. Do the angles at the vertices are right angles?
3. Can you identify the figure?

4.3 Angle Bisector

(i) Activity : Paper folding

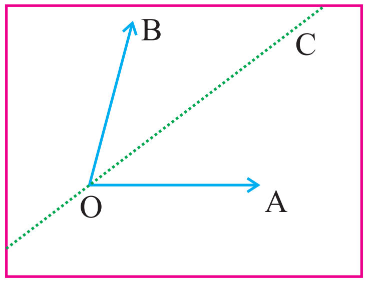


- Take a sheet of paper and mark a point O on it. With O as initial point draw two rays OA and OB to make ∠AOB.



- Fold the sheet through 'O' such that the rays OA and OB coincide with each other and make a crease on the paper.

- Let OC be the line of crease on the paper after unfold. By actual measurement, ∠AOC and ∠BOC are equal.
- So the line of crease OC divides the given angle into two equal parts.

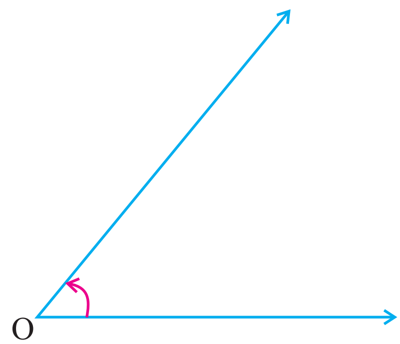


- This line of crease is the line of symmetry for ∠AOB.
- This line of symmetry for ∠AOB is called the angle bisector.

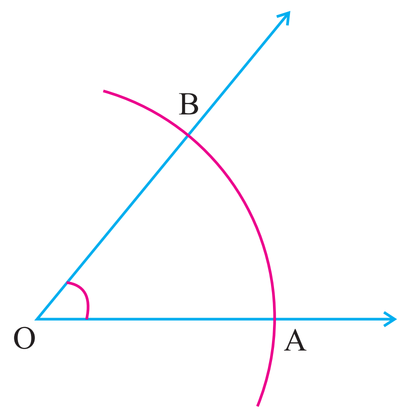
> The angle bisector of a given angle is the line of symmetry which divides the angle into two equal parts.

(ii) To construct an angle bisector of the given angle using scale and compass

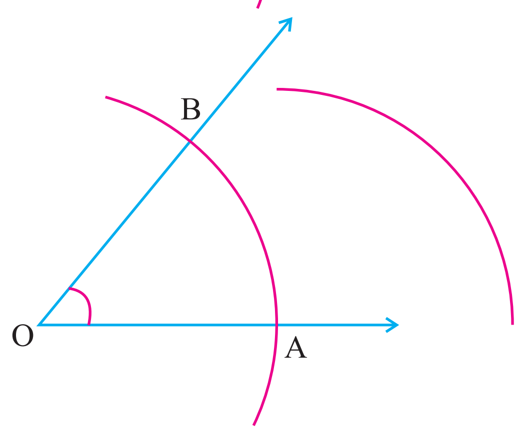
Step 1 : Construct an angle of given measure at O.



Step 2 : With 'O ' as centre draw an arc of any radius to cut the rays of the angle at A and B.

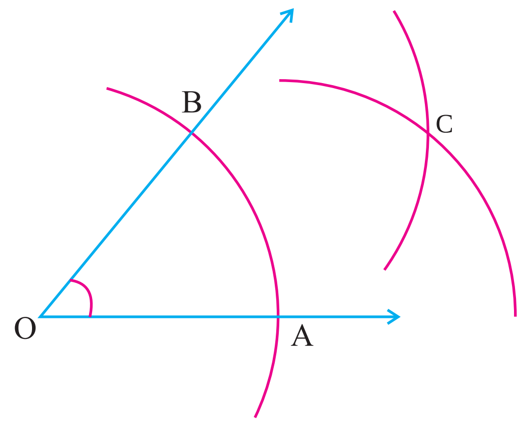


Step 3 : With 'A' as centre draw an arc of radius more than half of AB, in the interior of the given angle.



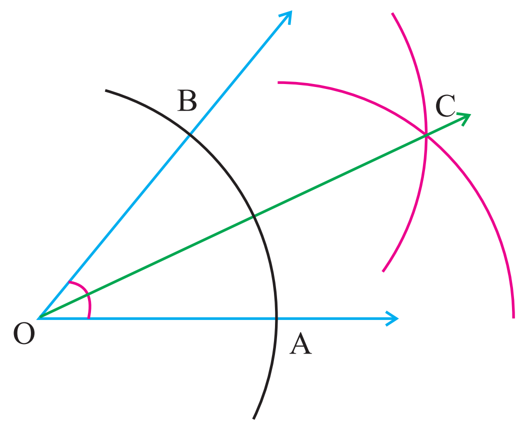
MATHEMATICS

Step 4 : With 'B' as centre draw an arc of same radius to cut the previous arc at 'C'.



Step 5 : Join OC.

OC is the angle bisector of the given angle.



Mark any point on the angle bisector OC. Verify that it is equidistant from the rays OA and OB.

Example 4.2

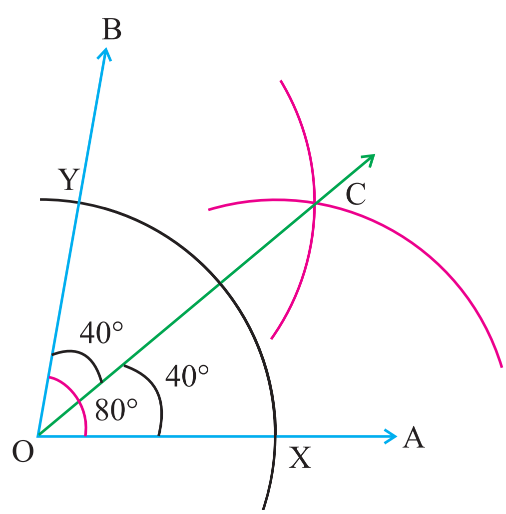
Construct ∠AOB = 80° and draw its angle bisector.

Solution

Step 1 : Construct ∠AOB = 80° angle at the point 'O' using protractor.

Step 2 : With 'O' as centre draw an arc of any radius to cut the rays OA and OB at the points X and Y respectively.

Step 3 : With 'X' as centre draw an arc of radius more than half of XY in the interior of the angle.



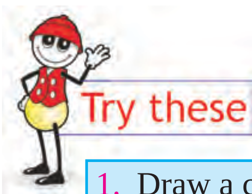
Step 4 : With 'Y' as centre draw an arc of the same radius to cut the previous arc at C. Join OC.

OC is the angle bisector of the given angle 80°.

Draw an angle of measure 120° and divide into four equal parts.

Exercise 4.1

1. Draw the line segment AB = 7cm and construct its perpendicular bisector.
2. Draw a line segment XY = 8.5 cm and find its axis of symmetry.
3. Draw a perpendicular bisector of the line segment AB = 10 cm.
4. Draw an angle measuring 70° and construct its bisector.
5. Draw an angle measuring 110° and construct its bisector.
6. Construct a right angle and bisect it using scale and compass.



1. Draw a circle with centre 'C' and radius 4 cm. Draw any chord AB. Construct perpendicular bisector to AB and examine whether it passes through the centre of the circle.
2. Draw perpendicular bisectors to any two chords of equal length in a circle. (i) Where do they meet? (ii) Verify whether the chords are at a same distance from the centre.
3. Plot three points not on a straight line. Find a point equidistant from them.

Hint: Join all the points in order. You get a triangle. Draw perpendicular bisectors to each side. They meet at a point which is equidistant from the points you have plotted. This point is called circumcentre.

5

DATA HANDLING

5.1 Introduction

Data Handling is a part of statistics. The word statistics is derived from the Latin word " Status". Like Mathematics, Statistics is also a science of numbers. The numbers referred to here are data expressed in numerical form like,

(i) Marks of students in a class

(ii) Weight of children of particular age in a village

(iii) The amount of rainfall in a region over a period of years.

Statistics deals with the methods of collection, classification, analysis and interpretation of such data.

Any collection of information in the form of numerical figures giving the required information is called data.

Raw data

The marks obtained in Mathematics test by the students of a class is a collection of observations gathered initially. The information which is collected initially and presented randomly is called a raw data.

The raw data is an unprocessed and unclassified data.

Grouped data

Some times the collected raw data may be huge in number and it gives us no information as such. Whenever the data is large, we have to group them meaningfully and then analyse.

The data which is arranged in groups or classes is called a grouped data.

Collection of data

The initial step of investigation is the collection of data. The collected data must be relevant to the need.

Primary data

Try these

Find the relevant data for the students from tribal villages are good visual learners.

For example, Mr. Vinoth, the class teacher of standard VII plans to take his students for an excursion. He asks the students to give their choice for

(i) particular location they would like to go

(ii) the game they would like to play

(iii) the food they would like to have on their trip

For all these, he is getting the information directly from the students. This type of collection of data is known as primary data.

5.2 Collecting and Organizing of Continuous Data

Secondary data

Mr. Vinoth, the class teacher of standard VII is collecting the information about weather for their trip. He may collect the information from the internet, news papers, magazines, television and other sources. These external sources are called secondary data.

Variable

As far as statistics is concerned the word variable means a measurable quantity which takes any numerical value within certain limits.

Few etxamples are (i) age, (ii) income, (iii) height and (iv) weight.

Frequency

Suppose we measure the height of students in a school. It is possible that a particular value of height say 140 cm gets repeated. We then count the number of times the value occurs. This number is called the frequency of 140 cm.

The number of times a particular value repeats itself is called its frequency.

Range

The difference between the highest value and the lowest value of a particular data is called the range.

Example 5.1

Let the heights (in cm) of 20 students in a class be as follows.

120, 122, 127, 112, 129, 118, 130, 132, 120, 115

124, 128, 120, 134, 126, 110, 132, 121, 127, 118.

Here the least value is 110 cm and the highest value is 134 cm.

Range = Highest value - Lowest value

$= 134 - 110 = 24$

Class and Class Interval

In the above example if we take 5 classes say 110 - 115, 115 - 120, 120 - 125, 125 - 130, 130 - 135 then each class is known as class interval. The class interval must be of equal size. The number of classes is neither too big nor too small. i.e The optimum number of classes is between 5 and 10.

Class limits

In class 110 - 115, 110 is called the lower limit of the class and 115 is the upper limit of the class.

Width (or size) of the class interval:

The difference between the upper and lower limit is called the width of the class interval. In the above example, the width of the class interval is 115 - 110 = 5. By increasing the class interval, we can reduce the number of classes.

There are two types of class intervals. They are (i) Inclusive form and (ii) Exclusive form.

(i) Inclusive form

In this form, the lower limit as well as upper limit will be included in that class interval. For example, in the first class interval 110 - 114, the heights 110 as well as 114 are included. In the second class interval 115 - 119, both the heights 115 and 119 are included and so on.

(ii) Exclusive form

In the above example 5.1, in the first class interval 110 - 115, 110 cm is included and 115 cm is excluded. In the second class interval 115 is included and 120 is excluded and so on. Since the two class intervals contain 115 cm, it is customary to include 115 cm in the class interval 115 - 120, which is the lower limit of the class interval.

Tally marks

In the above example 5.1, the height 110 cm, 112 cm belongs to the class interval 110 - 115. We enter | | tally marks. Count the tally marks and enter 2 as the frequency in the frequency column.

If five tally marks are to be made we mark four tally marks first and the fifth one one is marked across, so that ~~||||~~ represents a cluster of five tally marks.

To represent seven, we use a cluster of five tally marks and then add two more tally marks as shown ~~||||~~ ||.

Frequency Table

A table which represents the data in the form of three columns, first column showing the variable (Number) and the second column showing the values of the variable (Tally mark) and the third column showing their frequencies is called a **frequency table** (Refer table 5.3).

If the values of the variable are given using different classes and the frequencies are marked against the respective classes, we get a frequency distribution. All the frequencies are added and the number is written as the total frequency for the entire intervals. This must match the total number of data given. The above process of forming a frequency table is called tabulation of data.

Now we have the following table for the above data. (Example 5.1)

Inclusive form

Class Interval	Tally Marks	Frequency
110 - 114	||	2
115 - 119	|||	3
120 - 124	~~||||~~ |	6
125 - 129	~~||||~~	5
130 - 134	||||	4
	Total	20

Table 5.1

Exclusive form

Class Interval	Tally Marks	Frequency
110 - 115	||	2
115 - 120	|||	3
120 - 125	~~||||~~ |	6
125 - 130	~~||||~~	5
130 - 135	||||	4
	Total	20

Table 5.2

Frequency table for an ungrouped data

Example 5.2

Construct a frequency table for the following data.

5, 1, 3, 4, 2, 1, 3, 5, 4, 2

1, 5, 1, 3, 2, 1, 5, 3, 3, 2.

Solution

From the data, we observe the numbers 1, 2, 3, 4 and 5 are repeated. Hence under the number column, write the five numbers 1, 2, 3, 4, and 5 one below the other.

Now read the number and put the tally mark in the tally mark column against the number. In the same way put the tally mark till the last number. Add the tally marks against the numbers 1, 2, 3, 4 and 5 and write the total in the corresponding frequency column. Now, add all the numbers under the frequency column and write it against the total.

Number	Tally Marks	Frequency
1	卌	5
2	ǀǀǀǀ	4
3	卌	5
4	ǀǀ	2
5	ǀǀǀǀ	4
	Total	20

Table 5.3

In the formation of Frequency distribution for the given data values, we should

(i) select a suitable number of classes, not very small and also not very large.

(ii) take a suitable class - interval or class width and

(iii) present the classes with increasing values without any gaps between classes.

Frequency table for a grouped data

Example 5.3

The following data relate to mathematics marks obtained by 30 students in standard VII. Prepare a frequency table for the data.

25, 67, 78, 43, 21, 17, 49, 54, 76, 92, 20, 45, 86, 37, 35

60, 71, 49, 75, 49, 32, 67, 15, 82, 95, 76, 41, 36, 71, 62

Solution:

The minimum marks obtained is 15.

The maximum marks obtained is 95.

Range = Maximum value – Minimum value

= 95 – 15

= 80

Choose 9 classes with a class interval of 10. as 10 - 20, 20 - 30,⋯,90 - 100. The following is the frequency table.

Class Interval (Marks)	Tally Marks	Frequency
10 - 20	||	2
20 - 30	|||	3
30 - 40	||||	4
40 - 50	~~||||~~	5
50 - 60	||	2
60 - 70	||||	4
70 - 80	~~||||~~ |	6
80 - 90	||	2
90 - 100	||	2
	Total	30

Table 5.4

5.2 Continuous grouped Frequency distribution Table

To find the class limits in continuous grouped frequency distribution.

Steps to do

(i) Find the difference between the upper limit of the first class and lower limit of the second class.

(ii) Divide the difference by 2. Let the answer be x.

(iii) Subtract 'x' from lower limits of all the class intervals.

(iv) Add 'x' to all the upper limits of all the class intervals. Now the new limits will be true class limits.

Example 5.4

Form the frequency distribution table for the following data which gives the ages of persons who watched a particular channel on T.V.

Class Interval (Age)	10 -19	20 -29	30 - 39	40 - 49	50 - 59	60 - 69
Number of persons	45	60	87	52	25	12

Solution:

In this table, the classes given here have gaps. Hence we rewrite the classes using the exclusive method.

Difference between upper limits of first class and lower limits of second class

$$= 20 - 19 = 1$$

Divide the difference by 2 then,

$$x = \frac{1}{2} = 0.5$$

Now subtract 0.5 from lower limits and add 0.5 to the upper limits. Now we get continuous frequency distribution table with true class limits.

Class Interval (Age)	Frequency (Number of persons)
9.5 - 19.5	45
19.5 - 29.5	60
29.5 - 39.5	87
39.5 - 49.5	52
49.5 - 59.5	25
59.5 - 69.5	12

Table 5.5

Exercise 5.1

1. Choose the correct answer :

i) The difference between the highest and lowest value of the variable in the given data. is called.

(A) Frequency (B) Class limit (C) Class interval (D) Range

ii) The marks scored by a set of students in a test are 65, 97, 78, 49, 23, 48, 59, 98. The range for this data is

(A) 90 (B) 74 (C) 73 (D) 75

iii) The range of the first 20 natural numbers is

(A) 18 (B) 19 (C) 20 (D) 21

iv) The lower limit of the class interval 20 - 30 is

(A) 30 (B) 20 (C) 25 (D) 10

v) The upper of the class interval 50 - 60 is

(A) 50 (B) 60 (C) 10 (D) 55

2. Construct a frequency table for each of the following data:

   10, 15, 13, 12, 14, 11, 11, 12, 13, 15

   11, 13, 12, 15, 13, 12, 14, 14, 15, 11

3. In the town there were 26 patients in a hospital.

   The number of tablets given to them is given below. Draw a frequency table for the data.

   2, 4, 3, 1, 2, 2, 2, 4, 3, 5, 2, 1, 1, 2

   4, 5, 1, 2, 5, 4, 3, 3, 2, 1, 5, 4.

4. The number of savings book accounts opened in a bank during 25 weeks are given as below. Form a frequency table for the data:

   15, 25, 22, 20, 18, 15, 23, 17, 19, 12, 21, 26, 30

   19, 17, 14, 20, 21, 24, 21, 16, 22, 20, 17, 14

5. The weight (in kg) 20 persons are given below.

   42, 45, 51, 55, 49, 62, 41, 52, 48, 64

   52, 42, 49, 50, 47, 53, 59, 60, 46, 54

   Form a frequency table by taking class intervals 40 - 45, 45 - 50, 50 - 55, 55 - 60 and 60 - 65.

6. The marks obtained by 30 students of a class in a mathematics test are given below.

   45, 35, 60, 41, 8, 28, 31, 39, 55, 72, 22, 75, 57, 33, 51

   76, 30, 49, 19, 13, 40, 88, 95, 62, 17, 67, 50, 66, 73, 70

   Form a grouped frequency table:

7. Form a continuous frequency distribution table from the given data.

Class Interval (weight in kg.)	21 - 23	24 - 26	27 - 29	30 - 32	33 - 35	36 - 38
Frequency (Number of children)	2	6	10	14	7	3

8. The following data gives the heights of trees in a grove. Form a continuous frequency distribution table.

Class Interval (Height in metres)	2 - 4	5 - 7	8 - 10	11 - 13	14 - 16
Frequency (Number of trees)	29	41	36	27	12

Points to Remember

1. Any collection of information in the form of numerical figures giving the required information is called data.
2. The raw data is an unprocessed and unclassified data.
3. The data which is arranged in groups (or classes) is called a grouped data.
4. The number of times a particular value repeats itself is called its frequency.
5. Range = Highest value – Lowest value.
6. The difference between the upper and the lower limit is called the width of the class interval.

ANSWERS

Chapter - 1

Exercise 1.1

1. i) D	ii) B	iii) C	iv) B		
2. i) 0	ii) – 5	iii) 5	iv) 0		
3. i) – 6	ii) – 25	iii) 651	iv) – 316	v) 0	vi) 1320
vii) 25	viii) 25	ix) 42	x) – 24	xi) 1890	xii) – 1890
xiii) –1440	xiv) 256	xv) 6000	xvi) 10800		
4. i) – 135	ii) 16	iii) 182	iv) – 800	v) 1	vi) 0
5. ₹ 645	6. 75 marks		7. ₹1500		8. ₹240

Exercise 1.2

1. i) D	ii) A	iii) C	iv) A		
2. i) – 5	ii) 10	iii) 4	iv) – 1	v) – 6	vi) – 9
vii) – 1	viii) 2	ix) 2	x) 6		
3. i) 20	ii) 20	iii) – 400			
4. – 5					

Exercise 1.3

1. i) $\frac{24}{5}$	ii) $\frac{9}{7}$	iii) 2	iv) 3	v) $\frac{14}{3}$	vi) 20
vii) $\frac{77}{4}$	viii) 10	ix) 8	x) 24		
2. i) 14	ii) 63	iii) 16	iv) 25	v) 288	vi) 16
vii) 9	viii) 70	ix) 25	x) 50		
3. i) $26\frac{1}{4}$	ii) $19\frac{4}{5}$	iii) $9\frac{3}{5}$	iv) $64\frac{2}{7}$	v) $52\frac{1}{2}$	vi) $85\frac{1}{2}$

4. Vasu drank 4 litres.

Exercise 1.4

1. i) 1	ii) $\frac{7}{12}$	iii) $\frac{7}{12}$	iv) $\frac{7}{18}$	v) 1	vi) $\frac{2}{63}$
2. i) $\frac{22}{27}$	ii) $\frac{1}{5}$	iii) $\frac{1}{4}$	iv) $\frac{9}{16}$	v) $\frac{9}{2}$	vi) $\frac{48}{35}$
3. i) $2\frac{4}{15}$	ii) $4\frac{29}{40}$	iii) $7\frac{1}{2}$	iv) $20\frac{1}{8}$	v) $59\frac{13}{16}$	

4. 55 km

5. $12\frac{1}{4}$ hrs

Exercise 1.5

1. i) $\frac{7}{5}$ ii) $\frac{9}{4}$ iii) $\frac{7}{10}$ iv) $\frac{4}{9}$ v) $\frac{2}{33}$ vi) 9
   vii) 13 viii) $\frac{5}{7}$
2. i) $\frac{1}{15}$ ii) $\frac{1}{54}$ iii) $\frac{1}{6}$ iv) $\frac{1}{12}$
3. i) $\frac{8}{5}$ ii) $\frac{35}{36}$ iii) $4\frac{7}{12}$ iv) $1\frac{11}{16}$
4. 21 uniforms
5. 40 km/hour

Exercise 1.6

1. i) A ii) C iiii) B iv) D
2. i) $\frac{-20}{15}, \frac{-19}{15}, \frac{-18}{15}, \frac{-17}{15}$ ii) $\frac{7}{6}, \frac{6}{6}, \frac{5}{6}, \frac{4}{6}$
   iii) $\frac{48}{28}, \frac{47}{28}, \frac{46}{28}, \frac{45}{28}$
3. i) $\frac{-3}{4}$ ii) $\frac{-3}{8}$ iii) $\frac{-3}{5}$ iv) $\frac{-5}{3}$ v) $\frac{-1}{2}$
5. i, iv, v

Exercise 1.7

1. i) C ii) C iii) D iv) D
2. i) $\frac{18}{5}$ ii) $\frac{24}{13}$ iii) 2 iv) $\frac{-12}{13}$ v) $\frac{13}{3}$ vi) $\frac{19}{42}$
   vii) $\frac{-43}{21}$ viii) -3 ix) $\frac{24}{7}$ x) $\frac{-13}{30}$
3. i) 1 ii) 4 iii) $\frac{-9}{44}$ iv) $\frac{-5}{16}$ v) $\frac{23}{20}$ vi) -1
   vii) $\frac{-69}{26}$ viii) $\frac{-41}{60}$ ix) $\frac{-1}{27}$ x) $\frac{1}{12}$
4. i) $\frac{2}{35}$ ii) $\frac{1}{4}$ iii) $\frac{19}{12}$ iv) $\frac{3}{2}$ v) $\frac{-43}{28}$
5. i) $4\frac{7}{11}$ ii) $-3\frac{1}{2}$ iii) $1\frac{7}{11}$ iv) $5\frac{3}{4}$ v) $-1\frac{17}{40}$ vi) $-4\frac{7}{132}$
   vii) $-6\frac{41}{42}$ viii) $-3\frac{7}{210}$
6. $\frac{7}{4}$
7. $\frac{4}{5}$
8. $13\frac{17}{20}$ kg.
9. $18\frac{3}{4}$ kg.
10. $3\frac{9}{10}$ kg.

Answers

Exercise 1.8

1. i) C ii) B iii) A iv) A

2. i) $\frac{-72}{25}$ ii) $\frac{-35}{169}$ iii) $\frac{-7}{24}$ iv) $\frac{-12}{11}$ v) -20 vi) $\frac{2}{9}$

3. i) $\frac{-15}{4}$ ii) -5 iii) $26\frac{98}{125}$ iv) $66\frac{44}{375}$ v) $\frac{45}{28}$

4. i) $\frac{16}{81}$ ii) $\frac{-3}{2}$ iii) $\frac{-8}{7}$ iv) $-9\frac{3}{43}$

5. $\frac{9}{7}$ 6. $\frac{3}{2}$

Exercise 1.9

1. i) C ii) C iii) A iv) C

2. i) 2.1 ii) 40.5 iii) 17.1 iv) 82.8 v) 0.45 vi) 1060.15
   vii) 2.58 viii) 1.05 ix) 10.34 x) 1.041 xi) 4.48 xii) 0.00125
   xiii) 2.108 xiv) 0.0312

3. i) 14 ii) 468 iii) 4567 iv) 2690.8 v) 3230 vi) 17140
   vi) 478

4. 51.5 cm^2 5. 756 km.

Exercise 1.10

1. i) A ii) B iii) C iv) B

2. i) 0.3 ii) 0.09 iii) 1.16 iv) 10.8 v) 196.3 vi) 3.04

3. i) 0.68 ii) 4.35 iii) 0.09 iv) 4.43 v) 37.348 vi) 0.079

4. i) 0.056 ii) 0.007 iii) 0.0069 iv) 7.436 v) 0.437 vi) 0.7873

5. i) 0.0089 ii) 0.0733 iii) 0.04873
   iv) 0.1789 v) 0.0009 vi) 0.00009

6. i) 2 ii) 160 iii) 12.5 iv) 8.19 v) 2 vi) 35

7. 23 km 8. 10.5 kg 9. 9Books 10. 42.2 km/hour 11. 14.4

Exercise 1.11

1. i) A ii) A iii) C iv) C

2. i) 256 ii) 27 iii) 1331 iv) 1728 v) 28561 vi) 0

3. i) 7^6 ii) 1^5 iii) 0^6 iv) b^5 v) 2^2a^4 vi) $(1003)^3$

4. i) $2^3 \times 3^3$ ii) 3^5 iii) 5^4 iv) 2^{10} v) 5^5 vi) 10^5

5. i) 4^5 ii) 2^6 iii) 3^2 iv) 5^6 v) 2^7 vi) 4^7

6. i) $5^2 \times 2^2$ ii) $2^7 \times 3^1$ iii) $2^1 \times 3^1 \times 133^1$ iv) $2^1 \times 3^1 \times 113^1$
   v) $2^2 \times 3 \times 79$ vi) $2^7 \times 5^1$
7. i) 200000 ii) 0 iii) 2025 iv) 1296
   v) 9000000000 vi) 0
8. i) – 125 ii) 1 iii) 72 iv) – 2000 v) 10584 vi) – 131072

Exercise 1.12

1. i) A ii) A iii) C iv) C
2. i) 3^{12} ii) a^{12} iii) 7^{5+x} iv) 10^7 v) 5^9
3. i) 5^4 ii) a^4 iii) 10^{10} iv) 4^2 v) $3^0 = 1$
4. i) 3^{12} ii) 2^{20} iii) 2^{20} iv) 1 v) 5^{20}

Chapter - 2

Exercise 2.1

1. (i) A (ii) D (iii) D (iv) B (v) C
2. Constants: 5, – 9.5; Variables: a, $-xy$, p.
3. (i) $x + 6$ (ii) $-m - 7$ (iii) $3q + 11$ (iv) $3x + 10$ (v) $5y - 8$
4. 3, – 4, 9
5. (i) $y^2 x$, coefficient = y^2. (ii) x, coefficient = 1.
   (iii) zx, coefficient = z. (iv) $-5xy^2$, coefficient = $-5y^2$.
6. (i) $-my^2$, coefficient = $-m$. (ii) $6y^2$, coefficient = 6.
   (iii) $-9xy^2$, coefficient = $-9x$.

Exercise 2.2

1. (i) B (ii) D (iii) D (iv) D (v) A
2. (i) $4x, 7x$ (ii) $7b, -3b$ (iii) $3x^2y, -8yx^2$ (iv) $a^2b, 7a^2b$
   (v) $5pq, 25pq$; $-4p, 10p$; $3q, 70q$; $p^2q^2, 14p^2q^2$
3. (i) 2 (ii) 2 (iii) 3 (iv) 4 (v) 2
4. (i) – 10 (ii) 10 (iii) 11
5. (i) 21 (ii) 34 (iii) 82

Exercise 2.3

1. (i) C (ii) B (iii) A (iv) D (v) A
2. (i) $13a + 2b$ (ii) $5l - 4l^2$ (iii) $16z^2 - 16z$
   (iv) $p - q$ (v) $7m^2n - 4m^2 - 6n^2 + 4mn^2$ (vi) $x^2 - 3xy + 7y^2$

3. (i) $2ab$ (ii) $2s + t$ (iii) $3a – 2b + 2p + 3q$
   (iv) $5a – 5b + 4$ (v) $2x + 2y – 2$
   (vi) $7c + 4$ (vii) $3m^2n + 5mn – 4n^2 + 4$
4. (i) $8a$ (ii) $7a^2b$ (iii) $– 11x^2y^2$ (iv) $– 2xy + 16$
   (v) $5n – 2mn + 3m$ (vi) $– 5p – 15p^2$ (vii) $8m^2 – 6m – 12$
   (viii) $s^2 – 6s – 4$ (ix) $9n^2 – 10mn – 9m^2$
5. (i) $x^2 + 5xy – 3y^2$ (ii) $9p – 2q – 6$ (iii) $4x – 3y + 9$
6. $6a – 6$
7. $16x + 12$
8. ₹$12a – 2$
9. $7x – 8$ metres
10. (i) $8p^2 – 9p – 11$ (ii) $– p^2 + 8p + 12$
11. $2m^2 + 5m + 10$

Chapter - 3

Exercise 3.1

1. (i) B (ii) C (iii) A (iv) C (v) A
2. (i) Equilateral triangle - 3 lines of symmetry (iv) Rhombus - 2 lines of symmetry
5. (i) isosceles triangle (ii) equilateral triangle (iii) scalene triangle

Exercise 3.2

1. (i) C (ii) B (iii) D (iv) B (v) D
2. (i) 90° (ii) 90° (iii) 180° (iv) 180°
3. (i) 90°, 4 (ii) 360°, 1 (iii) 180°, 2 (iv) 360°, 1
4. 45°, 8

Exercise 3.3

1. (i) A (ii) B (iii) C (iv) D (v) D
2. (i) ∠DOC, ∠COB; ∠COB, ∠BOA
   (ii) ∠QOX, ∠XOP; ∠POY, ∠YOQ; ∠YOQ, ∠QOX; ∠XOP, ∠POY
3. ∠POR, ∠QOS; ∠SOP, ∠ROQ
4. (i) 150° (ii) 100°
   (iii) 110° (iv) 120° (v) 135°

5. ∠BOC = 145°; ∠AOD = 145°; ∠COA = 35°.

6. (i) 80° (ii) 110°

(iii) 20° (iv) 80°

(v) 36° (vi) 45°

7. $y = 120°$; $x = 60°$ 8. $x = 25°$

Chapter - 5

Exercise 5.1

1. (i) D (ii) D (iii) B (iv) B (v) B