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TRUTH ALONE TRIUMPHS

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STANDARD EIGHT

TERM I

MATHEMATICS

SCIENCE

SOCIAL SCIENCE

Volume 2

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MATHEMATICS

STANDARD EIGHT

TERM I

Volume 2

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1 Real Number System

Paul Erdos
[26 March, 1913 - 20 September, 1996]

He was a great prolific and notable Hungarian mathematician. Erdos published more papers than any other mathematician in history, working with hundreds of collaborators in many fields including number theory.

His fascination with mathematics developed early at the age of three. He could calculate how many seconds a person had lived. His life was documented in the film "*N is a Number: A Portrait of Paul Erdos*", while he was still alive.

Erdos said, **"I know numbers are beautiful. If they aren't beautiful, nothing is."**

1.1 Introduction

Number theory as a fundamental body of knowledge has played a pivotal role in the development of Mathematics. The Greek Mathematician Pythagoras and his disciples believed that "everything is number" and that the central explanation of the universe lay in numbers.

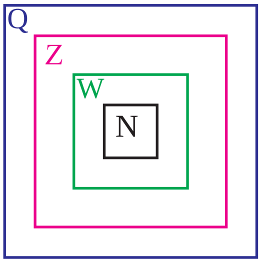
The system of writing numerals was developed some 10,000 years ago. India was the main centre for the development of the number system which we use today. It took about 5000 years for the complete development of the number system.

The Whole numbers are fountain head of all Mathematics. The present system of writing numerals is known as Hindu-Arabic numeral system.

In this system, we use the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. It is also called the decimal system with base 10. The word 'decimal' comes from Latin word 'Decem' which means 'Ten'.

Mathematics is the 'Queen of Science' and
Number theory is the 'Queen of Mathematics'.

In class VII, we have learnt about Natural numbers N = {1, 2, 3, ⋯}, Whole numbers W = {0, 1, 2, ⋯}, Integers Z = {⋯, – 2, – 1, 0, 1, 2, ⋯} and Rational numbers Q and also the four fundamental operations on them.



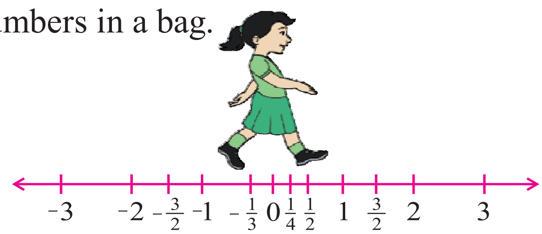
State whether the following statements are True Or False
a) All Integers are Rational Numbers.
b) All Natural Numbers are Integers.
c) All Integers are Natural Numbers.
d) All Whole Numbers are Natural Numbers.
e) All Natural Numbers are Whole Numbers.
f) All Rational Numbers are Whole Numbers.

1.2 Revision : Representation of Rational Numbers on the Number Line

Rational numbers

The numbers of the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$ are known as rational numbers. The collection of numbers of the form $\frac{p}{q}$, where $q > 0$ is denoted by Q. Rational numbers include natural numbers, whole numbers, integers and all negative and positive fractions.

Here we can visualize how the girl collected all the rational numbers in a bag.



Rational numbers can also be represented on the number line and here we can see a picture of a girl walking on the number line.

Encircle the correct type of Number system

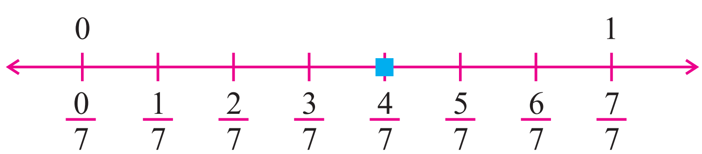
Number	Number System			
4	N	W	Z	Q
-6	N	W	Z	Q
5/3	N	W	Z	Q
0	N	W	Z	Q
$\sqrt{9}$	N	W	Z	Q
$\sqrt[3]{8}$	N	W	Z	Q
34.7	N	W	Z	Q

To express rational numbers appropriately on the number line, divide each unit length into as many number of equal parts as the denominator of the rational number and then mark the given number on the number line.

Illustration:

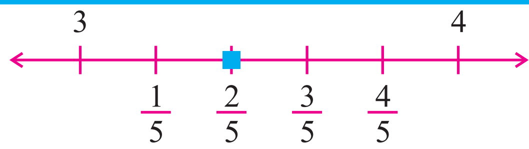
(i) Express $\frac{4}{7}$ on the number line.

$\frac{4}{7}$ lies between 0 and 1.



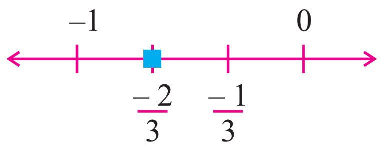
(ii) $\frac{17}{5} = 3\frac{2}{5}$

It lies between 3 and 4.



(iii) $-\frac{2}{3}$

It lies between – 1 and 0.



Every natural number is a rational number. Is the converse true?

1.3 Four Properties of Rational Numbers

1.3.1 (a) Addition

(i) Closure property

The sum of any two rational numbers is always a rational number. This is called 'Closure property of addition' of rational numbers. Thus, Q is closed under addition.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d}$ is also a rational number.

Illustration: (i) $\frac{2}{9} + \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$ is a rational number.

(ii) $5 + \frac{1}{3} = \frac{5}{1} + \frac{1}{3} = \frac{15+1}{3} = \frac{16}{3} = 5\frac{1}{3}$ is a rational number.

(ii) Commutative property

Addition of two rational numbers is commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$.

Illustration: For two rational numbers $\frac{1}{2}, \frac{2}{5}$ we have

$$\frac{1}{2} + \frac{2}{5} = \frac{2}{5} + \frac{1}{2}$$

LHS $= \frac{1}{2} + \frac{2}{5}$	RHS $= \frac{2}{5} + \frac{1}{2}$
$= \frac{5+4}{10} = \frac{9}{10}$	$= \frac{4+5}{10} = \frac{9}{10}$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Commutative property is true for addition.

(iii) Associative property

Addition of rational numbers is associative.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f}$.

Illustration: For three rational numbers $\frac{2}{3}, \frac{1}{2}$ and 2, we have

$$\frac{2}{3}+\left(\frac{1}{2}+2\right) = \left(\frac{2}{3}+\frac{1}{2}\right)+2$$

$$\text{LHS} = \frac{2}{3}+\left(\frac{1}{2}+2\right) = \frac{2}{3}+\left(\frac{1}{2}+\frac{2}{1}\right) = \frac{2}{3}+\left(\frac{1}{2}+\frac{4}{2}\right) = \frac{2}{3}+\frac{5}{2} = \frac{4+15}{6} = \frac{19}{6} = 3\frac{1}{6}$$

$$\text{RHS} = \left(\frac{2}{3}+\frac{1}{2}\right)+2 = \left(\frac{4}{6}+\frac{3}{6}\right)+2 = \frac{7}{6}+2 = \frac{7}{6}+\frac{2}{1} = \frac{7+12}{6} = \frac{19}{6} = 3\frac{1}{6}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Associative property is true for addition.

(iv) Additive identity

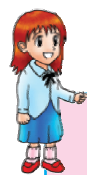
The sum of any rational number and zero is the rational number itself.

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b}+0=\frac{a}{b}=0+\frac{a}{b}$.

Zero is the additive identity for rational numbers.

Illustration: (i) $\frac{2}{7}+0=\frac{2}{7}=0+\frac{2}{7}$

(ii) $\left(\frac{-7}{11}\right)+0=\frac{-7}{11}=0+\left(\frac{-7}{11}\right)$



Do you know?

Zero is a special rational number. It can be written as $0=\frac{0}{q}$ where $q \neq 0$.

(v) Additive inverse

$\left(\frac{-a}{b}\right)$ is the negative or additive inverse of $\frac{a}{b}$.

If $\frac{a}{b}$ is a rational number, then there exists a rational number $\left(\frac{-a}{b}\right)$ such that $\frac{a}{b}+\left(\frac{-a}{b}\right)=0$.

Illustration: (i) Additive inverse of $\frac{3}{5}$ is $\frac{-3}{5}$

(ii) Additive inverse of $\frac{-3}{5}$ is $\frac{3}{5}$

(iii) Additive inverse of 0 is 0 itself.

Try these

Numbers	Addition		
	Closure property	Commutative property	Associative property
Natural numbers			
Whole numbers			Yes
Integers			
Rational numbers	Yes		

1.3.1 (b) Subtraction

(i) Closure Property

The difference between any two rational numbers is always a rational number. Hence Q is closed under subtraction.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d}$ is also a rational number.

Illustration: (i) $\frac{4}{7} - \frac{2}{7} = \frac{2}{7}$ is a rational number.

(ii) $1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$ is a rational number.

(ii) Commutative Property

Subtraction of two rational numbers is not commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d} \neq \frac{c}{d} - \frac{a}{b}$.

Illustration: For two rational numbers $\frac{4}{9}$ and $\frac{2}{5}$, we have

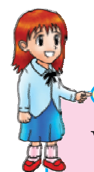
$$\frac{4}{9} - \frac{2}{5} \neq \frac{2}{5} - \frac{4}{9}$$

$$\text{LHS} = \frac{4}{9} - \frac{2}{5} = \frac{20-18}{45} = \frac{2}{45}$$

$$\text{RHS} = \frac{2}{5} - \frac{4}{9} = \frac{18-20}{45} = \frac{-2}{45}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$ Commutative property is not true for subtraction.



Do you know?

When two rational numbers are equal, then commutative property is true for them.

(iii) Associative property

Subtraction of rational numbers is not associative.

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} - \left(\frac{c}{d} - \frac{e}{f}\right) \neq \left(\frac{a}{b} - \frac{c}{d}\right) - \frac{e}{f}$.

Illustration: For three rational numbers $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, we have

$$\frac{1}{2} - \left(\frac{1}{3} - \frac{1}{4}\right) \neq \left(\frac{1}{2} - \frac{1}{3}\right) - \frac{1}{4}$$

$$\text{LHS} = \frac{1}{2} - \left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{2} - \left(\frac{4-3}{12}\right) = \frac{1}{2} - \left(\frac{1}{12}\right) = \frac{6-1}{12} = \frac{5}{12}$$

$$\text{RHS} = \left(\frac{1}{2} - \frac{1}{3}\right) - \frac{1}{4} = \left(\frac{3-2}{6}\right) - \frac{1}{4} = \frac{1}{6} - \frac{1}{4} = \frac{2-3}{12} = \frac{-1}{12}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$ Associative property is not true for subtraction.

Try these

Numbers	Subtraction		
	Closure property	Commutative property	Associative property
Natural numbers	No		
Whole numbers			
Integers			
Rational numbers			No

1.3.1 (c) Multiplication

(i) Closure property

The product of two rational numbers is always a rational number. Hence Q is closed under multiplication.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$ is also a rational number.

Illustration: (i) $\frac{1}{3}\times 7=\frac{7}{3}=2\frac{1}{3}$ is a rational number.

(ii) $\frac{4}{3}\times\frac{5}{9}=\frac{20}{27}$ is a rational number.

(ii) Commutative property

Multiplication of rational numbers is commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}\times\frac{c}{d}=\frac{c}{d}\times\frac{a}{b}$.

Illustration: For two rational numbers $\frac{3}{5}$ and $\frac{-8}{11}$, we have

$$\frac{3}{5}\times\left(\frac{-8}{11}\right) = \left(\frac{-8}{11}\right)\times\frac{3}{5}$$

$$\text{LHS}=\frac{3}{5}\times\left(\frac{-8}{11}\right)=\frac{-24}{55} \qquad \text{RHS}=\frac{-8}{11}\times\left(\frac{3}{5}\right)=\frac{-24}{55}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Commutative property is true for multiplication.

(iii) Associative property

Multiplication of rational numbers is associative.

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)=\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f}$.

Illustration: For three rational numbers $\frac{1}{2}, \left(\frac{-1}{4}\right)$ and $\frac{1}{3}$, we have

$$\frac{1}{2}\times\left(\frac{-1}{4}\times\frac{1}{3}\right) = \left(\frac{1}{2}\times\left(\frac{-1}{4}\right)\right)\times\frac{1}{3}$$

$$\text{LHS} = \frac{1}{2}\times\left(\frac{-1}{12}\right) = \frac{-1}{24} \quad\Big|\quad \text{RHS} = \left(\frac{-1}{8}\right)\times\frac{1}{3} = \frac{-1}{24}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Associative property is true for multiplication.

(iv) Multiplicative identity

The product of any rational number and 1 is the rational number itself. 'One' is the multiplicative identity for rational numbers.

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b}\times 1 = \frac{a}{b} = 1\times\frac{a}{b}$.

Illustration: (i) $\frac{5}{7}\times 1 = \frac{5}{7}$

(ii) $\left(\frac{-3}{8}\right)\times 1 = \frac{-3}{8}$

Think it!

Is 1 the multiplicative identity for integers?

(v) Multiplication by 0

Every rational number multiplied with 0 gives 0.

If $\frac{a}{b}$ is any rational number, then $\frac{a}{b}\times 0 = 0 = 0\times\frac{a}{b}$.

Illustration: (i) $-5\times 0 = 0$

(ii) $\left(\frac{-7}{11}\right)\times 0 = 0$

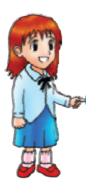
(vi) Multiplicative Inverse or Reciprocal

For every rational number $\frac{a}{b}$, $a\neq 0$, there exists a rational number $\frac{c}{d}$ such that $\frac{a}{b}\times\frac{c}{d} = 1$. Then $\frac{c}{d}$ is called the multiplicative inverse of $\frac{a}{b}$.

If $\frac{a}{b}$ is a rational number, then $\frac{b}{a}$ is the multiplicative inverse or reciprocal of it.

Illustration: (i) The reciprocal of 2 is $\frac{1}{2}$.

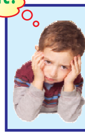
(ii) The multiplicative inverse of $\left(\frac{-3}{5}\right)$ is $\left(\frac{-5}{3}\right)$.



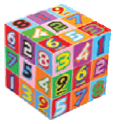
Do you know?

i) 0 has no reciprocal.
ii) 1 and – 1 are the only rational numbers which are their own reciprocals.

Think it!



Is 0.3 the reciprocal of $3\frac{1}{3}$?



Try these

Numbers	Multiplication		
	Closure property	Commutative property	Associative property
Natural numbers			
Whole numbers		Yes	
Integers			Yes
Rational numbers			

1.3.1 (d) Division

(i) Closure property

The collection of non-zero rational numbers is closed under division.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, such that $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d}$ is always a rational number.

Illustration: (i) $\frac{2}{3} \div \frac{1}{3} = \frac{2}{3} \times \frac{3}{1} = \frac{2}{1} = 2$ is a rational number.

(ii) $\frac{4}{5} \div \frac{3}{2} = \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$ is a rational number.

(ii) Commutative property

Division of rational numbers is not commutative.

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} \div \frac{c}{d} \neq \frac{c}{d} \div \frac{a}{b}$

Illustration: For two rational numbers $\frac{4}{5}$ and $\frac{3}{8}$, we have

$$\frac{4}{5} \div \frac{3}{8} \neq \frac{3}{8} \div \frac{4}{5}$$

$$\text{LHS} = \frac{4}{5} \times \frac{8}{3} = \frac{32}{15} \qquad \text{RHS} = \frac{3}{8} \times \frac{5}{4} = \frac{15}{32}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$ Commutative property is not true for division.

(iii) Associative property

Division of rational numbers is not associative.

If $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \div \left(\frac{c}{d} \div \frac{e}{f}\right) \neq \left(\frac{a}{b} \div \frac{c}{d}\right) \div \frac{e}{f}$.

Illustration: For three rational numbers $\frac{3}{4}$, 5 and $\frac{1}{2}$, we have

$$\frac{3}{4} \div \left(5 \div \frac{1}{2}\right) \neq \left(\frac{3}{4} \div 5\right) \div \frac{1}{2}$$

$$\text{LHS} = \frac{3}{4} \div \left(5 \div \frac{1}{2}\right) \qquad \text{RHS} = \left(\frac{3}{4} \div 5\right) \div \frac{1}{2}$$

$$= \frac{3}{4} \div \left(\frac{5}{1} \times \frac{2}{1}\right) \qquad = \left(\frac{3}{4} \times \frac{1}{5}\right) \div \frac{1}{2}$$

$$= \frac{3}{4} \div 10 \qquad = \frac{3}{20} \times \frac{2}{1}$$

$$= \frac{3}{4} \times \frac{1}{10} = \frac{3}{40} \qquad = \frac{3}{10}$$

$$\therefore \text{LHS} \neq \text{RHS}$$

$\therefore$ Associative property is not true for division.

Try these

Numbers	Division		
	Closure property	Commutative property	Associative property
Natural numbers	No		
Whole numbers			
Integers			
Rational numbers		No	

1.3.1 (e) Distributive Property

(i) Distributive property of multiplication over addition

Multiplication of rational numbers is distributive over addition.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$.

Illustration: For three rational numbers $\frac{2}{3}, \frac{4}{9}$ and $\frac{3}{5}$, we have

$$\frac{2}{3} \times \left(\frac{4}{9} + \frac{3}{5}\right) = \frac{2}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{3}{5}$$

$$\text{LHS} = \frac{2}{3} \times \left(\frac{4}{9} + \frac{3}{5}\right) \qquad \text{RHS} = \frac{2}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{3}{5}$$

$$= \frac{2}{3} \times \left(\frac{20+27}{45}\right) \qquad = \frac{8}{27} + \frac{2}{5}$$

$$= \frac{2}{3} \times \frac{47}{45} = \frac{94}{135} \qquad = \frac{40+54}{135} = \frac{94}{135}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Multiplication is distributive over addition.

(ii) Distributive property of multiplication over subtraction

Multiplication of rational numbers is distributive over subtraction.

If $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$ are any three rational numbers, then $\frac{a}{b} \times \left(\frac{c}{d} - \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} - \frac{a}{b} \times \frac{e}{f}$.

Illustration: For three rational numbers $\frac{3}{7}$, $\frac{4}{5}$ and $\frac{1}{2}$, we have

$$\frac{3}{7}\times\left(\frac{4}{5}-\frac{1}{2}\right) = \frac{3}{7}\times\frac{4}{5}-\frac{3}{7}\times\frac{1}{2}$$

$$\text{LHS} = \frac{3}{7}\times\left(\frac{4}{5}-\frac{1}{2}\right) = \frac{3}{7}\times\left(\frac{8-5}{10}\right) = \frac{3}{7}\times\frac{3}{10} = \frac{9}{70}$$

$$\text{RHS} = \frac{3}{7}\times\frac{4}{5}-\frac{3}{7}\times\frac{1}{2} = \frac{12}{35}-\frac{3}{14} = \frac{24-15}{70} = \frac{9}{70}$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ Multiplication is distributive over subtraction.

EXERCISE 1.1

1. Choose the correct answer:

 i) The additive identity of rational numbers is ________ .

 (A) 0 (B) 1 (C) – 1 (D) 2

 ii) The additive inverse of $\frac{-3}{5}$ is ________ .

 (A) $\frac{-3}{5}$ (B) $\frac{5}{3}$ (C) $\frac{3}{5}$ (D) $\frac{-5}{3}$

 iii) The reciprocal of $\frac{-5}{13}$ is ________ .

 (A) $\frac{5}{13}$ (B) $\frac{-13}{5}$ (C) $\frac{13}{5}$ (D) $\frac{-5}{13}$

 iv) The multiplicative inverse of – 7 is ________ .

 (A) 7 (B) $\frac{1}{7}$ (C) – 7 (D) $\frac{-1}{7}$

 v) ________ has no reciprocal.

 (A) 0 (B) 1 (C) – 1 (D) $\frac{1}{4}$

2. Name the property under addition used in each of the following :

 (i) $\left(\frac{-3}{7}\right)+\frac{1}{9} = \frac{1}{9}+\left(\frac{-3}{7}\right)$ (ii) $\frac{4}{9}+\left(\frac{7}{8}+\frac{1}{2}\right) = \left(\frac{4}{9}+\frac{7}{8}\right)+\frac{1}{2}$

 (iii) $8+\frac{7}{10} = \frac{7}{10}+8$ (iv) $\left(\frac{-7}{15}\right)+0 = \frac{-7}{15} = 0+\left(\frac{-7}{15}\right)$

 (v) $\frac{2}{5}+\left(\frac{-2}{5}\right) = 0$

3. Name the property under multiplication used in each of the following:

 (i) $\frac{2}{3}\times\frac{4}{5} = \frac{4}{5}\times\frac{2}{3}$ (ii) $\left(\frac{-3}{4}\right)\times 1 = \frac{-3}{4} = 1\times\left(\frac{-3}{4}\right)$

(iii) $\left(\frac{-17}{28}\right)\times\left(\frac{-28}{17}\right) = 1$ (iv) $\frac{1}{5}\times\left(\frac{7}{8}\times\frac{4}{3}\right) = \left(\frac{1}{5}\times\frac{7}{8}\right)\times\frac{4}{3}$

(v) $\frac{2}{7}\times\left(\frac{9}{10}+\frac{2}{5}\right) = \frac{2}{7}\times\frac{9}{10}+\frac{2}{7}\times\frac{2}{5}$

4. Verify whether commutative property is satisfied for addition, subtraction, multiplication and division of the following pairs of rational numbers.

(i) 4 and $\frac{2}{5}$ (ii) $\frac{-3}{4}$ and $\frac{-2}{7}$

5. Verify whether associative property is satisfied for addition, subtraction, multiplication and division of the following pairs of rational numbers.

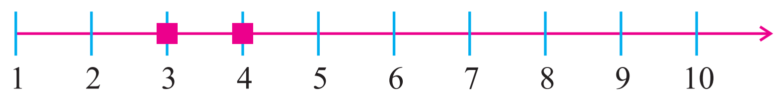
(i) $\frac{1}{3}, \frac{2}{5}$ and $\frac{-3}{7}$ (ii) $\frac{2}{3}, \frac{-4}{5}$ and $\frac{9}{10}$

6. Use distributive property of multiplication of rational numbers and simplify:

(i) $\frac{-5}{4}\times\left(\frac{8}{9}+\frac{5}{7}\right)$ (ii) $\frac{2}{7}\times\left(\frac{1}{4}-\frac{1}{2}\right)$

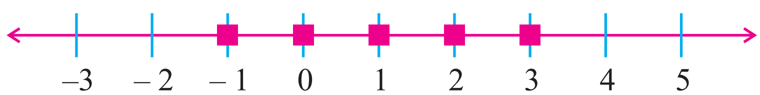
1.3.2 To find rational numbers between two rational numbers

Can you tell the natural numbers between 2 and 5?



They are 3 and 4.

Can you tell the integers between – 2 and 4?

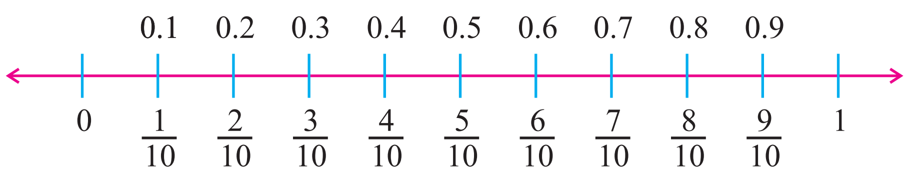


They are – 1, 0, 1, 2, 3.

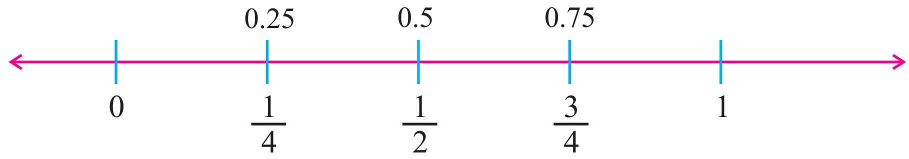
Now, Can you find any integer between 1 and 2?

No.

But, between any two integers, we have rational numbers.For example, between 0 and 1, we can find rational numbers $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \cdots$ which can be written as 0.1, 0.2, 0.3, $\cdots$.

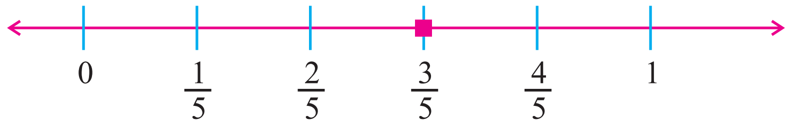


Similarly, we know that the numbers $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$ are lying between 0 and 1. These are rational numbers which can be written as 0.25, 0.5, 0.75 respectively.



Now, consider $\frac{2}{5}$ and $\frac{4}{5}$. Can you find any rational number between $\frac{2}{5}$ and $\frac{4}{5}$?

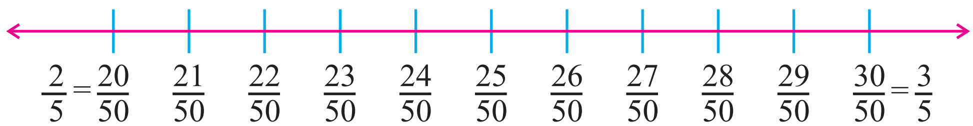
Yes. There is a rational number $\frac{3}{5}$.



In the same manner, we know that the numbers $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}$ and $\frac{4}{5}$ are lying between 0 and 1.

Can you find more rational numbers between $\frac{2}{5}$ and $\frac{3}{5}$?

Yes. We write $\frac{2}{5}$ as $\frac{20}{50}$ and $\frac{3}{5}$ as $\frac{30}{50}$, then we can find many rational numbers between them.

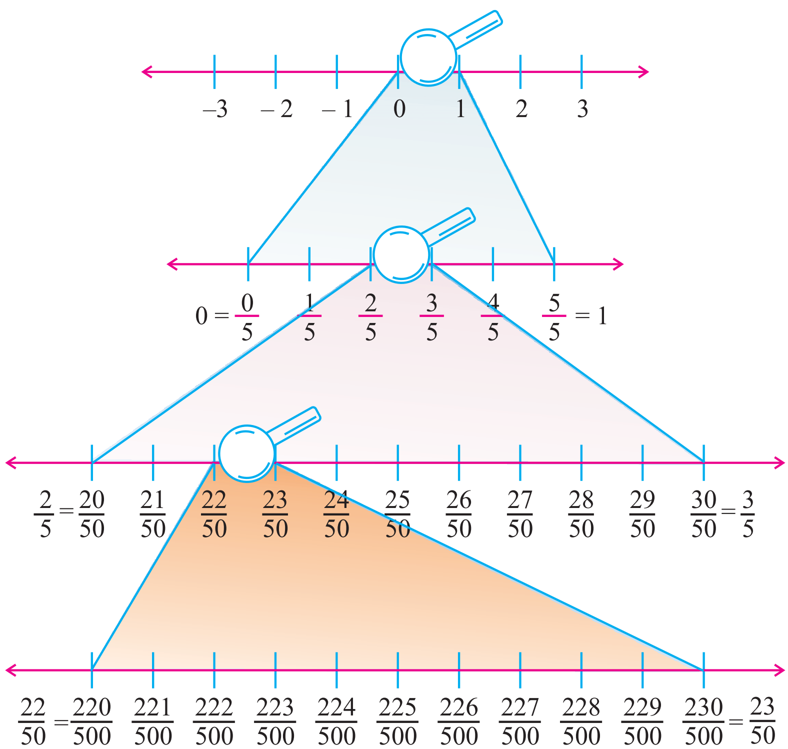


We can find nine rational numbers $\frac{21}{50}, \frac{22}{50}, \frac{23}{50}, \frac{24}{50}, \frac{25}{50}, \frac{26}{50}, \frac{27}{50}, \frac{28}{50}$ and $\frac{29}{50}$.

If we want to find some more rational numbers between $\frac{22}{50}$ and $\frac{23}{50}$, we write $\frac{22}{50}$ as $\frac{220}{500}$ and $\frac{23}{50}$ as $\frac{230}{500}$. Then we get nine rational numbers $\frac{221}{500}, \frac{222}{500}, \frac{223}{500}, \frac{224}{500}, \frac{225}{500}, \frac{226}{500}, \frac{227}{500}, \frac{228}{500}$ and $\frac{229}{500}$.

Let us understand this better with the help of the number line shown in the adjacent figure.

Observe the number line between 0 and 1 using a magnifying lens.



Similarily, we can observe many rational numbers in the intervals 1 to 2, 2 to 3 and so on.

If we proceed like this, we will continue to find more and more rational numbers between any two rational numbers. This shows that there is high density of rational numbers between any two rational numbers.

So, unlike natural numbers and integers, there are countless rational numbers between any two given rational numbers.

To find rational numbers between two rational numbers

We can find rational numbers between any two rational numbers in two methods.

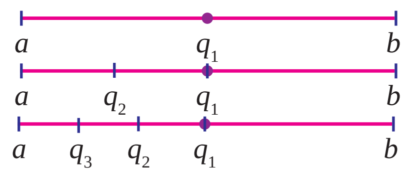
1. Formula method

Let 'a' and 'b' be any two given rational numbers. We can find many number of rational numbers $q_1, q_2, q_3, \ldots$ in between a and b as follows:

$$q_1 = \frac{1}{2}(a+b)$$

$$q_2 = \frac{1}{2}(a+q_1)$$

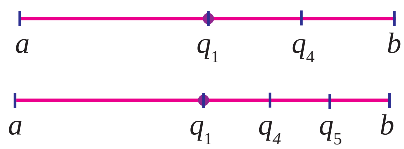
$$q_3 = \frac{1}{2}(a+q_2) \text{ and so on.}$$

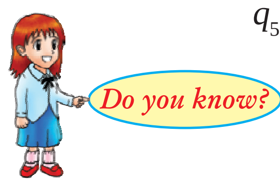


The numbers q_2, q_3 lie to the left of q_1. Similarly, q_4, q_5 are the rational numbers between 'a' and 'b' lie to the right of q_1 as follows:

$$q_4 = \frac{1}{2}(q_1+b)$$

$$q_5 = \frac{1}{2}(q_4+b) \text{ and so on.}$$





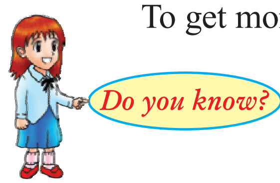
Average of two numbers always lie between them.

2. Aliter

Let 'a' and 'b' be two rational numbers.

(i) Convert the denominator of both the fractions into the same denominator by taking LCM. Now, if there is a number between numerators there is a rational number between them.

(ii) If there is no number between their numerators, then multiply their numerators and denominators by 10 to get rational numbers between them. To get more rational numbers, multiply by 100, 1000 and so on.



By following different methods one can get different rational numbers between 'a' and 'b'.

MATHEMATICS

Example 1.1

Find a rational number between $\frac{3}{4}$ and $\frac{4}{5}$.

Solution

Formula method:

Given: $a = \frac{3}{4}, b = \frac{4}{5}$

Let q_1 be the rational number between $\frac{3}{4}$ and $\frac{4}{5}$

$$q_1 = \frac{1}{2}(a+b)$$

$$= \frac{1}{2}\left(\frac{3}{4}+\frac{4}{5}\right) = \frac{1}{2}\left(\frac{15+16}{20}\right)$$

$$q_1 = \frac{1}{2}\times\left(\frac{31}{20}\right) = \frac{31}{40}$$

The rational number is $\frac{31}{40}$.

Aliter:

Given: $a = \frac{3}{4}, b = \frac{4}{5}$

We can write a and b as $\frac{3}{4}\times\frac{5}{5} = \frac{15}{20}$ and $\frac{4}{5}\times\frac{4}{4} = \frac{16}{20}$

To find a rational number between $\frac{15}{20}$ and $\frac{16}{20}$, we have to multiply the numerator and denominator by 10.

$$\frac{15}{20}\times\frac{10}{10} = \frac{150}{200}\ ,\quad \frac{16}{20}\times\frac{10}{10} = \frac{160}{200}$$

$\therefore$ The rational numbers between $\frac{150}{200}$ and $\frac{160}{200}$ are

$\frac{151}{200}, \frac{152}{200}, \frac{153}{200}, \frac{154}{200}, \frac{155}{200}, \frac{156}{200}, \frac{157}{200}, \frac{158}{200}$ and $\frac{159}{200}$.

Example 1.2

Find two rational numbers between $\frac{-3}{5}$ and $\frac{1}{2}$.

Solution

Given: $a = \frac{-3}{5}, b = \frac{1}{2}$

Let q_1 and q_2 be two rational numbers.

$$q_1 = \frac{1}{2}(a+b)$$

$$q_1 = \frac{1}{2}\times\left(\frac{-3}{5}+\frac{1}{2}\right) = \frac{1}{2}\times\left(\frac{-6+5}{10}\right) = \frac{1}{2}\times\left(\frac{-1}{10}\right) = \frac{-1}{20}$$

$$q_2 = \frac{1}{2}\ (a+q_1) = = \frac{1}{2}\times\left(\frac{-3}{5}+\left(\frac{-1}{20}\right)\right)$$

$$= \frac{1}{2}\times\left(\frac{-12+(-1)}{20}\right) = \frac{1}{2}\times\left(\frac{-12-1}{20}\right) = \frac{1}{2}\times\left(\frac{-13}{20}\right) = \frac{-13}{40}$$

The two rational numbers are $\frac{-1}{20}$ and $\frac{-13}{40}$.

Note: The two rational numbers can be inserted as $\frac{-3}{5} < \frac{-13}{40} < \frac{-1}{20} < \frac{1}{2}$

EXERCISE 1.2

1. Find one rational number between the following pairs of rational numbers.

 (i) $\frac{4}{3}$ and $\frac{2}{5}$ (ii) $\frac{-2}{7}$ and $\frac{5}{6}$ (iii) $\frac{5}{11}$ and $\frac{7}{8}$ (iv) $\frac{7}{4}$ and $\frac{8}{3}$

2. Find two rational numbers between

 (i) $\frac{2}{7}$ and $\frac{3}{5}$ (ii) $\frac{6}{5}$ and $\frac{9}{11}$ (iii) $\frac{1}{3}$ and $\frac{4}{5}$ (iv) $\frac{-1}{6}$ and $\frac{1}{3}$

3. Find three rational numbers between

 (i) $\frac{1}{4}$ and $\frac{1}{2}$ (ii) $\frac{7}{10}$ and $\frac{2}{3}$ (iii) $\frac{-1}{3}$ and $\frac{3}{2}$ (iv) $\frac{1}{8}$ and $\frac{1}{12}$

1.4 Simplification of Expressions Involving Three Brackets

Let us see some examples:

(i) $2 + 3 = 5$ (ii) $5 - 10 = -5$

(iii) $\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$ (iv) $4 - 2 \times \frac{1}{2} = ?$

In examples, (i), (ii) and (iii), there is only one operation. But in example (iv) we have two operations.

Do you know which operation has to be done first in problem (iv)?

In example (iv), if we do not follow some conventions, we will get different solutions.

For example (i) $(4-2) \times \frac{1}{2} = 2 \times \frac{1}{2} = 1$

(ii) $4 - \left(2 \times \frac{1}{2}\right) = 4 - 1 = 3$, we get different values.

So, to avoid confusion, certain conventions regarding the order of operations are followed. The operations are performed sequentially from left to right in the order of **'BODMAS'**.

B - brackets, **O** - of, **D** - division, **M** - multiplication, **A** - addition, **S** - subtraction.

Now we will study more about brackets and operation - of.

Brackets

Some grouping symbols are employed to indicate a preference in the order of operations. Most commonly used grouping symbols are given below.

Grouping symbols	Names
——	Bar bracket or Vinculum
( )	Parentheses or common brackets
{ }	Braces or Curly brackets
[ ]	Brackets or Square brackets

Operation - 'Of"

We sometimes come across expressions like 'twice of 3', 'one - fourth of 20', 'half of 10' etc. In these expressions, 'of' means 'multiplication with'.

For example,

(i) 'twice of 3' is written as 2×3,

(ii) 'one - fourth' of 20 is written as $\frac{1}{4} \times 20$,

(iii) 'half of 10' is written as $\frac{1}{2} \times 10$.

If more than one grouping symbols are used, we first perform the operations within the innermost symbol and remove it. Next we proceed to the operations within the next innermost symbols and so on.

Example 1.3

Simplify: $\left(1\frac{1}{3} + \frac{2}{3}\right) \times \frac{8}{15}$

Solution

$$\left(1\frac{1}{3} + \frac{2}{3}\right) \times \frac{8}{15} = \left(\frac{4}{3} + \frac{2}{3}\right) \times \frac{8}{15}$$

$$= \left(\frac{6}{3}\right) \times \frac{8}{15} \quad \text{[bracket is given preference]}$$

$$= 2 \times \frac{8}{15} = \frac{16}{15} = 1\frac{1}{15}.$$

Example 1.4

Simplify: $5\frac{1}{2} + \frac{3}{4}$ of $\frac{8}{9}$.

Solution

$$5\frac{1}{2} + \frac{3}{4} \text{ of } \frac{8}{9} = \frac{11}{2} + \frac{3}{4} \times \frac{8}{9} \quad \text{['of' is given preference]}$$

$$= \frac{11}{2} + \frac{24}{36} = \frac{11}{2} + \frac{2}{3}$$

$$= \frac{33+4}{6} = \frac{37}{6} = 6\frac{1}{6}.$$

Example 1.5

Simplify: $\left(\frac{-1}{3} \times \frac{5}{4}\right) + \left[\frac{3}{5} \div \left(\frac{1}{2} - \frac{1}{4}\right)\right]$

Solution

$$\left(\frac{-1}{3} \times \frac{5}{4}\right) + \left[\frac{3}{5} \div \left(\frac{1}{2} - \frac{1}{4}\right)\right] = \left(\frac{-1}{3} \times \frac{5}{4}\right) + \left[\frac{3}{5} \div \left(\frac{2-1}{4}\right)\right] \quad \text{[Innermost bracket is given preference]}$$

$$= \left(\frac{-1}{3} \times \frac{5}{4}\right) + \left[\frac{3}{5} \div \frac{1}{4}\right]$$

$$= \left(\frac{-1}{3} \times \frac{5}{4}\right) + \left[\frac{3}{5} \times 4\right] = \frac{-5}{12} + \frac{12}{5}$$

$$= \frac{-25+144}{60} = \frac{119}{60} = 1\frac{59}{60}.$$

Example 1.6

Simplify: $\frac{2}{7}-\left\{\left(\frac{1}{4}\div\frac{2}{3}\right)-\frac{5}{6}\right\}$

Solution

$$\begin{aligned}\frac{2}{7}-\left\{\left(\frac{1}{4}\div\frac{2}{3}\right)-\frac{5}{6}\right\} &= \frac{2}{7}-\left\{\left(\frac{1}{4}\times\frac{3}{2}\right)-\frac{5}{6}\right\}\\ &= \frac{2}{7}-\left\{\frac{3}{8}-\frac{5}{6}\right\}=\frac{2}{7}-\left\{\frac{9-20}{24}\right\}\\ &= \frac{2}{7}-\left\{\frac{-11}{24}\right\}=\frac{2}{7}+\frac{11}{24}\\ &= \frac{48+77}{168}=\frac{125}{168}.\end{aligned}$$

EXERCISE 1.3

1. Choose the correct answer:

(i) $2\times\frac{5}{3}=$ ________

(A) $\frac{10}{3}$ (B) $2\frac{5}{6}$ (C) $\frac{10}{6}$ (D) $\frac{2}{3}$

(ii) $\frac{2}{5}\times\frac{4}{7}=$ ________

(A) $\frac{14}{20}$ (B) $\frac{8}{35}$ (C) $\frac{20}{14}$ (D) $\frac{35}{8}$

(iii) $\frac{2}{5}+\frac{4}{9}$ is ________

(A) $\frac{10}{23}$ (B) $\frac{8}{45}$ (C) $\frac{38}{45}$ (D) $\frac{6}{13}$

(iv) $\frac{1}{5}\div 2\frac{1}{2}$ is ________

(A) $\frac{2}{25}$ (B) $\frac{1}{2}$ (C) $\frac{10}{7}$ (D) $\frac{3}{10}$

(v) $\left(1-\frac{1}{2}\right)+\left(\frac{3}{4}-\frac{1}{4}\right)$

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

2. Simplify:

(i) $\frac{11}{12}\div\left(\frac{5}{9}\times\frac{18}{25}\right)$

(ii) $\left(2\frac{1}{2}\times\frac{8}{10}\right)\div\left(1\frac{1}{2}+\frac{5}{8}\right)$

(iii) $\frac{15}{16}$ of $\left(\frac{5}{6}-\frac{1}{2}\right)\div\frac{10}{11}$

(iv) $\frac{9}{8}\div\frac{3}{5}$ of $\left(\frac{3}{4}+\frac{3}{5}\right)$

(v) $\frac{2}{5}\div\left\{\frac{1}{5}\text{ of }\left[\frac{3}{4}-\frac{1}{2}\right]-1\right\}$

(vi) $\left(1\frac{3}{4}\times 3\frac{1}{7}\right)-\left(4\frac{3}{8}\div 5\frac{3}{5}\right)$

(vii) $\left(\frac{1}{6}+2\frac{3}{4}\text{ of }1\frac{7}{11}\right)\div 1\frac{1}{6}$

(viii) $\left(\frac{-1}{3}\right)-\left\{1\div\left(\frac{2}{3}\times\frac{5}{7}\right)+8-\left[5-\overline{\frac{1}{2}-\frac{1}{4}}\right]\right\}$

1.5 Powers: Expressing the Numbers in Exponential Form with Integers as Exponent

In this section, we are going to study how to express the numbers in exponential form.

We can express $2 \times 2 \times 2 \times 2 = 2^4$, where 2 is the base and 4 is the index or power.

In general, a^n is the product of 'a' with itself n times, where 'a' is any real number and 'n' is any positive integer .'a' is called the base and 'n' is called the index or power.

Definition

If 'n' is a positive integer, then x^n means $\underbrace{x.x.x.....x}_{n \text{ factors}}$

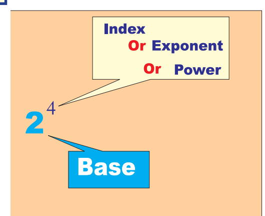
i.e, $x^n = \underbrace{x \times x \times x \times ..... \times x}_{n \text{ times}}$ (where 'n' is greater than 1)

Note : $x^1 = x$.

How to read?

7^3 is read as 7 raised to the power 3 (or) 7 cube.

Here 7 is called the base, 3 is known as **exponent** (or) **power** (or) **index**.



To illustrate this more clearly, let us look at the following table

S.No	Repeated multiplication of a number	Exponential form	Base	Power or Exponent or Index
1	$2 \times 2 \times 2 \times 2$	2^4	2	4
2	$(-4) \times (-4) \times (-4)$	$(-4)^3$	-4	3
3	$\left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right)$	$\left(\frac{2}{3}\right)^6$	$\frac{2}{3}$	6
4	$a \times a \times a \times ...$ m times	a^m	a	m

Example 1.7

Write the following numbers in powers of 2.

(i) 2 (ii) 8 (iii) 32 (iv) 128 (v) 256

Solution:

(i) $2 = 2^1$

(ii) $8 = 2 \times 2 \times 2 = 2^3$

(iii) $32 = 2\times2\times2\times2\times2 = 2^5$

(iv) $128 = 2\times2\times2\times2\times2\times2\times2 = 2^7$

(v) $256 = 2\times2\times2\times2\times2\times2\times2\times2 = 2^8$

1.6 Laws of Exponents with Integral Powers

With the above definition of positive integral power of a real number, we now establish the following properties called "laws of indices" or "laws of exponents".

(i) Product Rule

Law 1	$a^m \times a^n = a^{m+n}$, where 'a' is a real number and m, n are positive integers

Illustration

$$\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{3}\right)^4 = \left(\frac{2}{3}\right)^{3+4} = \left(\frac{2}{3}\right)^7$$ (Using the law, $a^m \times a^n = a^{m+n}$, where $a = \frac{2}{3}$, $m = 3$, $n = 4$)

(ii) Quotient Rule

Law 2	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$ and m, n are positive integers with $m > n$

Illustration

$$\frac{6^4}{6^2} = 6^{4-2} = 6^2$$ **(Using the law $\frac{a^m}{a^n} = a^{m-n}$, where a = 6,m=4,n=2)**

(iii) Power Rule

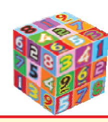
Law 3	$(a^m)^n = a^{m\times n}$, where m and n are positive integers

Illustration

$(3^2)^4 = 3^2\times3^2\times3^2\times3^2 = 3^{2+2+2+2} = 3^8$

we can get the same result by multiplying the two powers

i.e, $(3^2)^4 = 3^{2\times4} = 3^8$.



Try these

Show that $a^{(x-y)z} \times a^{(y-z)x} \times a^{(z-x)y} = 1$

(iv) Number with zero exponent

For $m \neq o$,

$m^3 \div m^3 = m^{3-3} = m^0$ (using law 2);

Aliter:

$m^3 \div m^3 = \frac{m^3}{m^3} = \frac{m\times m\times m}{m\times m\times m} = 1$

Using these two methods, $m^3 \div m^3 = m^0 = 1$.

From the above example, we come to the **fourth law** of exponent

Law 4	If 'a' is a rational number other than "zero", then $a^0 = 1$

Illustration

(i) $2^0 = 1$ (ii) $\left(\frac{3}{4}\right)^0 = 1$ (iii) $25^0 = 1$ (iv) $\left(-\frac{2}{5}\right)^0 = 1$ (v) $(-100)^0 = 1$

(v) Law of Reciprocal

The value of a number with negative exponent is calculated by converting into multiplicative inverse of the same number with positive exponent.

Illustration

(i) $4^{-4} = \frac{1}{4^4} = \frac{1}{4 \times 4 \times 4 \times 4} = \frac{1}{256}$

(ii) $5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}$

(iii) $10^{-2} = \frac{1}{10^2} = \frac{1}{10 \times 10} = \frac{1}{100}$

Reciprocal of 3 is equal to $\frac{1}{3} = \frac{3^0}{3^1} = 3^{0-1} = 3^{-1}$.

Similarly, reciprocal of $6^2 = \frac{1}{6^2} = \frac{6^0}{6^2} = 6^{0-2} = 6^{-2}$

Further, reciprocal of $\left(\frac{8}{3}\right)^3$ is equal to $\frac{1}{\left(\frac{8}{3}\right)^3} = \left(\frac{8}{3}\right)^{-3}$.

From the above examples, we come to the fifth law of exponent.

Law 5	If 'a' is a real number and 'm' is an integer, then $a^{-m} = \frac{1}{a^m}$

(vi) Multiplying numbers with same exponents

Consider the simplifications,

(i) $4^3 \times 7^3 = (4 \times 4 \times 4) \times (7 \times 7 \times 7) = (4 \times 7) \times (4 \times 7) \times (4 \times 7)$

$= (4 \times 7)^3$

(ii) $5^{-3} \times 4^{-3} = \frac{1}{5^3} \times \frac{1}{4^3} = \left(\frac{1}{5}\right)^3 \times \left(\frac{1}{4}\right)^3$

$= \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}$

$= \left(\frac{1}{5} \times \frac{1}{4}\right) \times \left(\frac{1}{5} \times \frac{1}{4}\right) \times \left(\frac{1}{5} \times \frac{1}{4}\right) = \left(\frac{1}{20}\right)^3$

$= 20^{-3} = (5 \times 4)^{-3}$

(iii) $\left(\frac{3}{5}\right)^2 \times \left(\frac{1}{2}\right)^2 = \left(\frac{3}{5} \times \frac{3}{5}\right) \times \left(\frac{1}{2} \times \frac{1}{2}\right) = \left(\frac{3}{5} \times \frac{1}{2}\right) \times \left(\frac{3}{5} \times \frac{1}{2}\right)$

$= \left(\frac{3}{5} \times \frac{1}{2}\right)^2$

In general, for any two integers a and b we have

$$a^2 \times b^2 = (a \times b)^2 = (ab)^2$$

$\therefore$ We arrive at the **power of a product rule** as follows:

$$(a \times a \times a \times \ldots m \text{ times}) \times (b \times b \times b \times \ldots m \text{ times}) = ab \times ab \times ab \times \ldots m \text{ times} = (ab)^m$$

(i.e.,) $a^m \times b^m = (ab)^m$

Law 6	$a^m \times b^m = (ab)^m$, where a, b are real numbers and m is an integer.

Illustration

(i) $3^x \times 4^x = (3\times 4)^x = 12^x$

(ii) $7^2 \times 2^2 = (7\times 2)^2 = 14^2 = 196$

(vii) Power of a quotient rule

Consider the simplifications,

(i) $\left(\frac{4}{3}\right)^2 = \frac{4}{3}\times\frac{4}{3} = \frac{16}{9} = \frac{4^2}{3^2}$ and

(ii) $\left(\frac{3}{5}\right)^{-2} = \frac{1}{\left(\frac{3}{5}\right)^2} = \frac{1}{\left(\frac{3^2}{5^2}\right)} = \frac{5^2}{3^2} = \left(\frac{5}{3}\right)^2 \quad \left(\because a^{-m} = \frac{1}{a^m}\right)$

$= \frac{5}{3}\times\frac{5}{3} = \frac{5\times 5}{3\times 3} = \frac{5^2}{3^2} = 5^2\times\frac{1}{3^2} = 5^2\times 3^{-2} = \frac{1}{5^{-2}}\times 3^{-2}$

$= \frac{3^{-2}}{5^{-2}}.$

Hence $\left(\frac{a}{b}\right)^2$ can be written as $\frac{a^2}{b^2}$

$$\left(\frac{a}{b}\right)^m = \left(\frac{a}{b}\times\frac{a}{b}\times\frac{a}{b}\times \ldots m \text{ times}\right) = \frac{a\times a\times a \ldots m \text{ times}}{b\times b\times b\times \ldots m\text{ times}}$$

$$\therefore \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Law 7	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$, a and b are real numbers, m is an integer

Illustration

(i) $\left(\frac{a}{b}\right)^7 = \frac{a^7}{b^7}$ (ii) $\left(\frac{5}{3}\right)^3 = \frac{5^3}{3^3} = \frac{125}{27}$

(iii) $\left(\frac{1}{4}\right)^4 = \frac{1^4}{4^4} = \frac{1}{256}$

Example 1.8

Simplify: (i) $2^5\times 2^3$ (ii) $10^9 \div 10^6$ (iii) $(x^0)^4$ (iv) $(2^3)^0$

(v) $\left(\frac{3}{2}\right)^5$ (vi) $(2^5)^2$ (vii) $(2\times 3)^4$

(viii) If $2^p = 32$, find the value of p.

Solution

(i) $2^5\times 2^3 = 2^{5+3} = 2^8$

(ii) $10^9 \div 10^6 = 10^{9-6} = 10^3$

(iii) $(x^0)^4 = (1)^4 = 1 \quad [\because a^0 = 1]$

(iv) $(2^3)^0 = 8^0 = 1 \quad [\because a^0 = 1]$

(v) $\left(\frac{3}{2}\right)^5 = \frac{3^5}{2^5} = \frac{243}{32}$

(vi) $(2^5)^2 = 2^{5\times 2} = 2^{10} = 1024$

(vii) $(2\times 3)^4 = 6^4 = 1296$

(or) $(2\times 3)^4 = 2^4\times 3^4 = 16\times 81 = 1296$

(viii) Given : $2^p = 32$

$2^p = 2^5$

Therefore $p = 5$ (Here the base on both sides are equal.)

$$\begin{array}{r|l} 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

Example 1.9

Find the value of the following:

(i) $3^4\times 3^{-3}$ (ii) $\frac{1}{3^{-4}}$ (iii) $\left(\frac{4}{5}\right)^2$ (iv) 10^{-3} (v) $\left(\frac{-1}{2}\right)^5$

(vi) $\left(\frac{7}{4}\right)^0\times 3$ (vii) $\left[\left(\frac{2}{3}\right)^2\right]^2$ (viii) $\left(\frac{3}{8}\right)^5\times\left(\frac{3}{8}\right)^4\div\left(\frac{3}{8}\right)^9$

Solution

(i) $3^4\times 3^{-3} = 3^{4+(-3)} = 3^{4-3} = 3^1 = 3$

(ii) $\frac{1}{3^{-4}} = 3^4 = 81$

(iii) $\left(\frac{4}{5}\right)^2 = \frac{4^2}{5^2} = \frac{16}{25}$

(iv) $10^{-3} = \frac{1}{1000}$

(v) $\left(\frac{-1}{2}\right)^5 = = \frac{-1}{32}$

(vi) $\left(\frac{7}{4}\right)^0\times 3 = 1\times 3 = 3$ $\left[\because \left(\frac{7}{4}\right)^0 = 1\right]$

(vii) $\left[\left(\frac{2}{3}\right)^2\right]^2 = \left(\frac{2}{3}\right)^{2\times 2} = \left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \frac{16}{81}$

(viii) $\left(\frac{3}{8}\right)^5\times\left(\frac{3}{8}\right)^4\div\left(\frac{3}{8}\right)^9 = \dfrac{\left(\frac{3}{8}\right)^{5+4}}{\left(\frac{3}{8}\right)^9} = \dfrac{\left(\frac{3}{8}\right)^9}{\left(\frac{3}{8}\right)^9} = 1$

(or) $\left(\frac{3}{8}\right)^{9-9} = \left(\frac{3}{8}\right)^0 = 1$

Example 1.10

Express 16^{-2} as a power with base 4.

Solution

We know that $16 = 4^2$

$\therefore 16^{-2} = (4^2)^{-2}$

$= 4^{2\times -2}$

$= 4^{-4}$

Example 1.11

Simplify

(i) $(2^3)^{-2} \times (3^2)^2$ (ii) $\dfrac{(2^2)^3}{(3^2)^2}$

Solution

(i) $(2^3)^{-2} \times (3^2)^2 = 2^{(3\times -2)} \times 3^{(2\times 2)}$

$= 2^{-6} \times 3^4 = \dfrac{1}{2^6} \times 3^4 = \dfrac{3^4}{2^6} = \dfrac{81}{64}$

(ii) $\dfrac{(2^2)^3}{(3^2)^2} = \dfrac{2^{2\times 3}}{3^{2\times 2}} = \dfrac{2^6}{3^4} = \dfrac{64}{81}.$

Example 1.12

Solve

(i) $12^x = 144$ (ii) $\left(\dfrac{2}{8}\right)^{2x} \times \left(\dfrac{2}{8}\right)^x = \left(\dfrac{2}{8}\right)^6$

Solution

(i) Given $12^x = 144$

$12^x = 12^2$

$\therefore\ x = 2$ ($\because$ The base on both sides are equal)

(ii) $\left(\dfrac{2}{8}\right)^{2x} \times \left(\dfrac{2}{8}\right)^x = \left(\dfrac{2}{8}\right)^6$

$\left(\dfrac{2}{8}\right)^{2x+x} = \left(\dfrac{2}{8}\right)^6$ ($\because$ The base on both sides are equal)

$2x + x = 6$

$3x = 6$

$x = \dfrac{6}{3} = 2.$

Example 1.13

Simplify: $\dfrac{(3^3)^{-2} \times (2^2)^{-3}}{(2^4)^{-2} \times 3^{-4} \times 4^{-2}}$

Solution

$$\frac{(3^3)^{-2} \times (2^2)^{-3}}{(2^4)^{-2} \times 3^{-4} \times 4^{-2}} = \frac{3^{-6} \times 2^{-6}}{2^{-8} \times 3^{-4} \times 4^{-2}}$$

$$= 3^{-6+4} \times 2^{-6+8} \times 4^2$$

$$= 3^{-2} \times 2^2 \times 4^2$$

$$= \frac{1}{3^2} \times 4 \times 16 = \frac{4 \times 16}{9}$$

$$= \frac{64}{9} = 7\frac{1}{9}.$$

EXERCISE 1.4

1. Choose the correct answer for the following:

(i) $a^m \times a^n$ is equal to

(A) $a^m + a^n$ (B) a^{m-n} (C) a^{m+n} (D) a^{mn}

(ii) p^0 is equal to

(A) 0 (B) 1 (C) – 1 (D) p

(iii) In 10^2, the exponent is

(A) 2 (B) 1 (C) 10 (D) 100

(iv) 6^{-1} is equal to

(A) 6 (B) – 1 (C) $-\frac{1}{6}$ (D) $\frac{1}{6}$

(v) The multiplicative inverse of 2^{-4} is

(A) 2 (B) 4 (C) 2^4 (D) – 4

(vi) $(-2)^{-5} \times (-2)^6$ is equal to

(A) – 2 (B) 2 (C) – 5 (D) 6

(vii) $(-2)^{-2}$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{-1}{2}$ (D) $\frac{-1}{4}$

(viii) $(2^0 + 4^{-1}) \times 2^2$ is equal to

(A) 2 (B) 5 (C) 4 (D) 3

(ix) $\left(\frac{1}{3}\right)^{-4}$ is equal to

(A) 3 (B) 3^4 (C) 1 (D) 3^{-4}

(x) $(-1)^{50}$ is equal to

(A) –1 (B) 50 (C) – 50 (D) 1

2. Simplify:

(i) $(-4)^5 \div (-4)^8$ (ii) $\left(\frac{1}{2^3}\right)^2$ (iii) $(-3)^4 \times \left(\frac{5}{3}\right)^4$

(iv) $\left(\frac{2}{3}\right)^5 \times \left(\frac{3}{4}\right)^2 \times \left(\frac{1}{5}\right)^2$ (v) $(3^{-7} \div 3^{10}) \times 3^{-5}$ (vi) $\frac{2^6 \times 3^2 \times 2^3 \times 3^7}{2^8 \times 3^6}$

(vii) $y^{a-b} \times y^{b-c} \times y^{c-a}$ (viii) $(4p)^3 \times (2p)^2 \times p^4$ (ix) $9^{5/2} - 3 \times 5^0 - \left(\frac{1}{81}\right)^{-1/2}$

(x) $\left(\frac{1}{4}\right)^{-2} - 3 \times 8^{2/3} \times 4^0 + \left(\frac{9}{16}\right)^{-1/2}$

3. Find the value of:

(i) $(3^0 + 4^{-1}) \times 2^2$ (ii) $(2^{-1} \times 4^{-1}) \div 2^{-2}$ (iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$

(iv) $(3^{-1} + 4^{-1} + 5^{-1})^0$ (v) $\left[\left(\frac{-2}{3}\right)^{-2}\right]^2$ (vi) $7^{-20} - 7^{-21}$.

MATHEMATICS

4. Find the value of m for which

(i) $5^m \div 5^{-3} = 5^5$ (ii) $4^m = 64$ (iii) $8^{m-3} = 1$

(iv) $(a^3)^m = a^9$ (v) $(5^m)^2 \times (25)^3 \times 125^2 = 1$ (vi) $2m = (8)^{\frac{1}{3}} \div (2^3)^{2/3}$

5. (a) If $2^x = 16$, find

(i) x (ii) $2^{\frac{x}{2}}$ (iii) 2^{2x} (iv) 2^{x+2} (v) $\sqrt{2^{-x}}$

(b) If $3^x = 81$, find

(i) x (ii) 3^{x+3} (iii) $3^{x/2}$ (iv) 3^{2x} (v) 3^{x-6}

6. Prove that (i) $\frac{3^{x+1}}{3^{x(x+1)}} \times \left(\frac{3^x}{3}\right)^{x+1} = 1$, (ii) $\left(\frac{x^m}{x^n}\right)^{m+n} . \left(\frac{x^n}{x^l}\right)^{n+l} . \left(\frac{x^l}{x^m}\right)^{l+m} = 1$

1.7 Squares, Square roots, Cubes and Cube roots

1.7.1 Squares

When a number is multiplied by itself we say that the number is squared. It is denoted by a number raised to the power 2.

For example : (i) $3 \times 3 = 3^2 = 9$

(ii) $5 \times 5 = 5^2 = 25$.

In example (ii) 5^2 is read as 5 to the power of 2 (or) 5 raised to the power 2 (or) 5 squared. 25 is known as the square of 5.

Similarly, 49 and 81 are the squares of 7 and 9 respectively.

In this section, we are going to learn a few methods of squaring numbers.

Perfect Square

The numbers 1, 4, 9, 16, 25, $\cdots$ are called **perfect squares or square numbers** as $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$ and so on.

A number is called a perfect square if it is expressed as the square of a number.

Properties of Square Numbers

We observe the following properties through the patterns of square numbers.

1. In square numbers, the digits at the unit's place are always 0, 1, 4, 5, 6 or 9. The numbers having 2, 3, 7 or 8 at its units' place are not perfect square numbers.

2. i)

Number	Square
1	1
9	81
11	121

If a number has 1 or 9 in the unit's place then its square ends in 1.

ii)

Number	Square
2	4
8	64
12	144

If a number has 2 or 8 in the unit's place then its square ends in 4.

iii)

Number	Square
3	9
7	49
13	169

If a number has 3 or 7 in the unit's place then its square ends in 9.

iv)

Number	Square
4	16
6	36
14	196

If a number has 4 or 6 in the unit's place then its square ends in 6.

v)

Number	Square
5	25
15	225
25	625

If a number has 5 in the unit's place then its square ends in 5.

3. Consider the following square numbers:

We have one zero

$$\left\{\begin{array}{l} 10^2 = 100 \\ 20^2 = 400 \\ 30^2 = 900 \end{array}\right\}$$

But we have two zeros

We have two zeros

$$\left\{\begin{array}{l} 100^2 = 10000 \\ 200^2 = 40000 \\ 700^2 = 490000 \end{array}\right\}$$

But we have four zeros

Result

(i) When a number ends with '0', its square ends with double zeros.

(ii) If a number ends with odd number of zeros then it is not a perfect square.

4. Consider the following:

(i) $100 = 10^2$

(Even number of zeros)

$\therefore$ 100 is a perfect square.

(ii) $81,000 = 81 \times 100 \times 10$

$= 9^2 \times 10^2 \times 10$ $\therefore$ 81,000 is not a perfect square.

(Odd number of zeros)

5. Observe the following tables:

Square of even numbers

Number	Square
2	4
4	16
6	36
8	64
10	100
⋮	⋮

Square of odd numbers

Number	Square
1	1
3	9
5	25
7	49
9	81
⋮	⋮

From the above table we infer that,

Result

(i) Squares of even numbers are even.

(ii) Squares of odd numbers are odd.

Example 1.14

Find the perfect square numbers between

(i) 10 and 20 (ii) 50 and 60 (iii) 80 and 90.

Solution

(i) The perfect square number between 10 and 20 is 16.

(ii) There is no perfect square number between 50 and 60.

(iii) The perfect square number between 80 and 90 is 81.

Example 1.15

By observing the unit's digits, which of the numbers 3136, 867 and 4413 can not be perfect squares?

Solution

Since 6 is in units place of 3136, there is a chance that it is a perfect square. 867 and 4413 are surely not perfect squares as 7 and 3 are the unit digit of these numbers.

Example 1.16

Write down the unit digits of the squares of the following numbers:

(i) 24 (ii) 78 (iii) 35

Solution

(i) The square of 24 = 24×24. Here 4 is in the unit place.

Therefore, we have 4×4 = 16. $\therefore$ 6 is in the unit digit of square of 24.

(ii) The square of 78 = 78 × 78. Here, 8 is in the unit place.

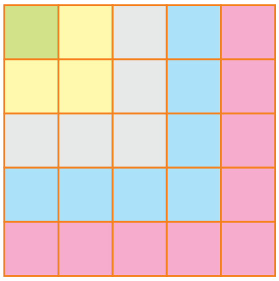
Therefore, we have 8 × 8 = 64. ∴ 4 is in the unit digit of square of 78

(iii) The square of 35 = 35 × 35. Here, 5 is in the unit place.

Therefore, we have 5 × 5 = 25. ∴ 5 is in the unit digit of square of 35.

Some interesting patterns of square numbers

Addition of consecutive odd numbers:

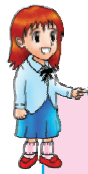


$$1 = 1 = 1^2$$
$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2$$
$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$
$$1 + 3 + 5 + 7 + \cdots + n = n^2 \text{ (sum of the first ‘}n\text{’ natural odd numbers)}$$

The above figure illustrates this result.

To find the square of a rational number $\frac{a}{b}$.

$$\frac{a}{b} \times \frac{a}{b} = \frac{a^2}{b^2} = \frac{\text{Square of the numerator}}{\text{Square of the denominator}}$$



Do you know?

(i) $45^2 = 2025 = (20+25)^2$

(ii) $55^2 = 3025 = (30 + 25)^2$

∴ 45, 55 are Kaprekar numbers

Illustration

(i) $\left(\frac{-3}{7}\right) \times \left(\frac{-3}{7}\right) = \left(\frac{-3}{7}\right)^2$

$$= \frac{(-3)\times(-3)}{7\times 7} = \frac{9}{49}$$

(ii) $\frac{5}{8} \times \frac{5}{8} = \left(\frac{5}{8}\right)^2 = \frac{25}{64}$.

EXERCISE 1.5

1. Just observe the unit digits and state which of the following are not perfect squares.

   (i) 3136 (ii) 3722 (iii) 9348 (iv) 2304 (v) 8343

2. Write down the unit digits of the following:

   (i) 78^2 (ii) 27^2 (iii) 41^2 (iv) 35^2 (v) 42^2

3. Find the sum of the following numbers without actually adding the numbers.

   (i) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15

   (ii) 1 + 3 + 5 + 7

   (iii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17

4. Express the following as a sum of consecutive odd numbers starting with 1

   (i) 7^2 (ii) 9^2 (iii) 5^2 (iv) 11^2

5. Find the squares of the following numbers

   (i) $\frac{3}{8}$ (ii) $\frac{7}{10}$ (iii) $\frac{1}{5}$ (iv) $\frac{2}{3}$ (v) $\frac{31}{40}$

6. Find the values of the following:

   (i) $(-3)^2$ (ii) $(-7)^2$ (iii) $(-0.3)^2$ (iv) $\left(-\frac{2}{3}\right)^2$ (v) $\left(-\frac{3}{4}\right)^2$ (vi) $(-0.6)^2$

7. Using the given pattern, find the missing numbers:

a)	b)
$1^2 + 2^2 + 2^2 = 3^2,$	$11^2 = 121$
$2^2 + 3^2 + 6^2 = 7^2$	$101^2 = 10201$
$3^2 + 4^2 + 12^2 = 13^2$	$1001^2 = 1002001$
$4^2 + 5^2 + \_\_\_ = 21^2$	$100001^2 = 1\_\_\_\_\_\_2\_\_\_\_\_\_1$
$5^2 + \_\_\_ + 30^2 = 31^2$	$10000001^2 = \_\_\_\_\_\_\_\_\_\_\_\_$
$6^2 + 7^2 + \_\_\_ = \_\_\_$	

1.7.2 Square roots

Definition

When a number is multiplied by itself, the product is called the square of that number. The number itself is called the **square root** of the product.

9 is the square of 3

$9 \rightleftarrows 3$

3 is the square root of 9

For example:

(i) $3 \times 3 = 3^2 = 9$

(ii) $(-3) \times (-3) = (-3)^2 = 9$

Here 3 and (-3) are the square roots of 9.

The symbol used for square root is $\sqrt{}$.

$\therefore \sqrt{9} = \pm 3$ (read as plus or minus 3)

Considering only the positive root, we have $\sqrt{9} = 3$

Note: We write the square root of x as $\sqrt{x}$ or $x^{\frac{1}{2}}$.

Hence, $\sqrt{4} = (4)^{\frac{1}{2}}$ and $\sqrt{100} = (100)^{\frac{1}{2}}$

In this unit, we shall take up only positive square root of a natural number. Observe the following table:

Table 1

Perfect Square	Square Root
1 16 36 81	1 4 6 9
100 225 2025 7396 9801	10 15 45 86 99
10,000 14,641 2,97,025 9,98,001	100 121 545 999
10,00,000 15,00,625 7,89,96,544 999,80,001	1000 1225 8888 9999

Single or double digit numeral has single digit in its square root.

3 or 4 digit numeral has 2 digits in its square root.

5 or 6 digit numeral has 3 digits in its square root.

7 or 8 digits numeral has 4 digits in its square root.

From the table, we can also infer that

(i) If a perfect square has 'n' digits where n is even, its square root has $\frac{n}{2}$ digits.

(ii) If a perfect square has 'n' digits where n is odd, its square root has $\frac{n+1}{2}$ digits.

To find a square root of a number, we have the following two methods.

(i) Factorization Method
(ii) Long Division Method

(i) Factorization Method

The square root of a perfect square number can be found by finding the prime factors of the number and grouping them in pairs.

Prime factorization

$$\begin{array}{r|l} 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

Example 1.17

Find the square root of 64

Solution

$$64 = \underbrace{2\times 2}\times\underbrace{2\times 2}\times\underbrace{2\times 2} = 2^2\times 2^2\times 2^2$$

$$\sqrt{64} = \sqrt{2^2\times 2^2\times 2^2} = 2\times 2\times 2 = 8$$

$$\sqrt{64} = 8$$

Example 1.18

Find the square root of 169

Solution

$$169 = \underbrace{13 \times 13} = 13^2$$

$$\sqrt{169} = \sqrt{13^2} = 13$$

Prime factorization

$$\begin{array}{r|l} 13 & 169 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

Example 1.19

Find the square root of 12.25

Solution

$$\sqrt{12.25} = \sqrt{\frac{12.25 \times 100}{100}}$$

$$= \frac{\sqrt{1225}}{\sqrt{100}} = \frac{\sqrt{5^2 \times 7^2}}{\sqrt{10^2}} = \frac{5 \times 7}{10}$$

$$\sqrt{12.25} = \frac{35}{10} = 3.5$$

Prime factorization

$$\begin{array}{r|l} 5 & 1225 \\ \hline 5 & 225 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

Example 1.20

Find the square root of 5929

Solution

$$5929 = \underbrace{7 \times 7} \times \underbrace{11 \times 11} = 7^2 \times 11^2$$

$$\sqrt{5929} = \sqrt{7^2 \times 11^2} = 7 \times 11$$

$$\therefore \sqrt{5929} = 77$$

Prime factorization

$$\begin{array}{r|l} 7 & 5929 \\ \hline 7 & 847 \\ \hline 11 & 121 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

Example 1.21

Find the least number by which 200 must be multiplied to make it a perfect square.

Solution

$$200 = 2 \times \underbrace{2 \times 2} \times \underbrace{5 \times 5}$$

'2' remains without a pair.

Hence, 200 must be multiplied by 2 to make it a perfect square.

Prime factorization

$$\begin{array}{r|l} 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Example 1.22

Find the least number by which 384 must be divided to make it a perfect square.

Solution

$$384 = 3 \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times 2$$

'3' and '2' remain without a pair.

Hence, 384 must be divided by 6 to make it a perfect square.

Prime factorization

$$\begin{array}{r|l} 3 & 384 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

(ii) Long division method

In case of large numbers, factors can not be found easily. Hence we may use another method, known as **Long division method**.

Using this method, we can also find square roots of decimal numbers. This method is explained in the following worked examples.

Example 1.23

Find the square root of 529 using long division method.

Solution

Step 1 : We write 529 as $5\ \overline{29}$ by grouping the numbers in pairs, starting from the right end. (i.e. from the unit's place).

Step 2 : Find the number whose square is less than (or equal to) 5. Here it is 2.

$$2\,\big|\,\overline{5\ \overline{29}}$$

Step 3 : Put '2' on the top, and also write 2 as a divisor as shown.

$$\begin{array}{r|l} & 2 \\ \hline 2 & 5\ \overline{29} \\ & 4 \\ \hline & 1 \end{array}$$

Step 4 : Multiply 2 on the top with the divisor 2 and write 4 under 5 and subtract. The remainder is 1.

Step 5 : Bring down the pair 29 by the side of the remainder 1, yielding 129.

$$\begin{array}{r|l} & 2 \\ \hline 2 & 5\ \overline{29} \\ & 4\ \downarrow \\ \hline & 1\ 29 \end{array}$$

Step 6 : Double 2 and take the resulting number 4. Find that number 'n' such that $4n \times n$ is just less than or equal to 129.

For example : $42 \times 2 = 84$; and $43 \times 3 = 129$ and so $n = 3$.

$$\begin{array}{r|l} & 2\ \ 3 \\ \hline 2 & 5\ \overline{29} \\ & 4\ \downarrow \\ \hline 43 & 1\ 29 \\ & 1\ 29 \\ \hline & \ \ 0 \end{array}$$

Step 7 : Write 43 as the next divisor and put 3 on the top along with 2. Write the product $43 \times 3 = 129$ under 129 and subtract. Since the remainder is '0', the division is complete.

Hence $\sqrt{529} = 23$.

Example 1.24

Find $\sqrt{3969}$ by the long division method.

Solution

Step 1 : We write 3969 as $\overline{39}\ \overline{69}$ by grouping the digits into pairs, starting from right end.

Step 2 : Find the number whose square is less than or equal to 39. It is 6.

Step 3 : Put 6 on the top and also write 6 as a divisor.

$$\begin{array}{r|l} & 6 \\ \hline 6 & \overline{39}\ \overline{69} \end{array}$$

Step 4 : Multiply 6 with 6 and write the result 36 under 39 and subtract. The remainder is 3.

$$\begin{array}{r|l} & 6 \\ \hline 6 & \overline{39}\ \overline{69} \\ & 36 \\ \hline & \ \ 3 \end{array}$$

Step 5 : Bring down the pair 69 by the side of this remainder 3, yielding 369.

$$\begin{array}{r|l} & 6 \\ \hline 6 & \overline{39}\ \overline{69} \\ & 36\ \downarrow \\ \hline & \ \ 3\ 69 \end{array}$$

Step 6 : Double 6, take the result 12 and find the number 'n'. Such that $12n \times n$ is just less than or equal to 369.

Since $122 \times 2 = 244;\ 123 \times 3 = 369,\ n = 3$

$$\begin{array}{r|l} & 6\ \ \ 3 \\ \hline 6 & \overline{39}\ \overline{69} \\ & 36\ \downarrow \\ \hline 123 & \ \ 3\ 69 \\ & \ \ 3\ 69 \\ \hline & \ \ \ \ \ 0 \end{array}$$

Step 7 : Write 123 as the next divisor and put 3 on the top along with 6. Write the product $123 \times 3 = 369$ under 369 and subtract. Since the remainder is '0', the division is complete.

Hence $\sqrt{3969} = 63$.

1.7.2 (a) Square roots of Decimal Numbers

To apply the long division method, we write the given number by pairing off the digits as usual in the integral part, and pairing off the digits in the decimal part from left to right after the decimal part.

For example, we write the number 322.48 as

322 • 48

Integral Part   Decimal Point   Fractional Part

We should know how to mark the decimal point in the square root. For this we note that for a number with 1 or 2 digits, the square root has 1 digit and so on. (Refer Table 1). The following worked examples illustrate this method:

Example 1.25

Find the square root of 6.0516

Solution

We write the number as $6\,.\,\overline{05}\;\overline{16}$. Since the number of digits in the integral part is 1, the square root will have 1 digit in its integral part. We follow the same procedure that we usually use to find the square root of 60516

$$
\begin{array}{r|l}
 & 2.\,4\;\;6 \\ \hline
2 & 6.\overline{05}\;\overline{16} \\
 & 4 \\ \hline
44 & 2\;05 \\
 & 1\;76 \\ \hline
486 & \;\;29\;16 \\
 & \;\;29\;16 \\ \hline
 & \;\;\;\;0
\end{array}
$$

From the above working, we get $\sqrt{6.0516} = 2.46$.

Example 1.26

Find the least number, which must be subtracted from 3250 to make it a perfect square

Solution

$$
\begin{array}{r|l}
 & 5\;\;7 \\ \hline
5 & \overline{32}\;\overline{50} \\
 & 25 \\ \hline
107 & \;7\;50 \\
 & \;7\;49 \\ \hline
 & \;\;\;\;1
\end{array}
$$

This shows that 57^2 is less than 3250 by 1. If we subtract the remainder from the number, we get a perfect square. So the required least number is 1.

Example 1.27

Find the least number, which must be added to 1825 to make it a perfect square.

Solution

$$
\begin{array}{r|l}
 & 4\;\;2 \\ \hline
4 & \overline{18}\;\overline{25} \\
 & 16 \\ \hline
82 & \;2\;25 \\
 & \;1\;64 \\ \hline
 & \;\;\;61
\end{array}
$$

This shows that $42^2 < 1825$.

Next perfect square is $43^2 = 1849$.

Hence, the number to be added is $43^2 - 1825 = 1849 - 1825$

$= 24.$

Example 1.28

Evaluate $\sqrt{0.182329}$

Solution

$$
\begin{array}{r|l}
 & 0.4\ \ 2\ \ 7 \\
\hline
4 & 0.\overline{18}\,\overline{23}\,\overline{29} \\
 & \ \ 16 \\
\hline
82 & \ \ \ 2\,23 \\
 & \ \ \ 1\,64 \\
\hline
847 & \ \ \ \ \ 59\,29 \\
 & \ \ \ \ \ 59\,29 \\
\hline
 & \ \ \ \ \ \ \ \ \ 0
\end{array}
$$

We write the number 0.182329 as $0.\overline{18}\,\overline{23}\,\overline{29}$. Since the number has no integral part, the square root also will have no integral part. We then proceed as usual for finding the square root of 182329.

Hence $\sqrt{0.182329} = 0.427$

Note: Since the integral part of the radicand is '0', the square root also has '0' in its integral part.

Example 1.29

Find the square root of 121.4404

Solution

$$
\begin{array}{r|l}
 & 1\ \ 1\ .\ 0\ \ 2 \\
\hline
1 & 1\,\overline{21}.\,\overline{44}\,\overline{04} \\
 & 1 \\
\hline
21 & 0\,21 \\
 & \ \ \ 21 \\
\hline
2202 & \ \ \ \ 0\ \ 44\,04 \\
 & \ \ \ \ \ \ \ 44\,04 \\
\hline
 & \ \ \ \ \ \ \ \ \ \ \ 0
\end{array}
$$

$\sqrt{121.4404} = 11.02$

Example 1.30

Find the square root of 0.005184

Solution

$\sqrt{0.005184} = 0.072$

$$
\begin{array}{r|l}
 & 0.\ 0\ \ 7\ \ 2 \\
\hline
7 & 0.\,\overline{00}\,\overline{51}\,\overline{84} \\
 & \ \ \ \ \ \ \ 49 \\
\hline
142 & \ \ \ \ \ \ \ \ 2\ 84 \\
 & \ \ \ \ \ \ \ \ 2\ 84 \\
\hline
 & \ \ \ \ \ \ \ \ \ \ \ 0
\end{array}
$$

Note: Since the integral part of the radicand is 0, a zero is written before the decimal point in the quotient. A '0' is written in the quotient after the decimal point since the first left period following the decimal point is 00 in the radicand.

1.7.2 (b) Square root of an Imperfect Square

An imperfect square is a number which is not a perfect square. For example 2, 3, 5, 7, 13,... are all imperfect squares. To find the square root of such numbers we use the Long division method.

If the required square root is to be found correct up to '*n*' decimal places, the square root is calculated up to n+1 decimal places and rounded to '*n*' decimal places. Accordingly, zeros are included in the decimal part of the radicand.

Example 1.31

Find the square root of 3 correct to two places of decimal.

Solution

$$
\begin{array}{r|l}
 & 1.\ 7\ \ 3\ \ 2 \\
\hline
1 & 3.\ \overline{00}\ \overline{00}\ \overline{00} \\
 & 1 \\
\hline
27 & 2\ 00 \\
 & 1\ 89 \\
\hline
343 & 11\ 00 \\
 & 10\ 29 \\
\hline
3462 & 71\ 00 \\
 & 69\ 24 \\
\hline
 & 1\ 76
\end{array}
$$

Since we need the answer correct to two places of decimal, we shall first find the square root up to three places of decimal. For this purpose we must add 6 (that is three pairs of) zeros to the right of the decimal point.

$\therefore \sqrt{3} \ = \ 1.732$ up to three places of decimal.

$\sqrt{3} \ = \ 1.73$ correct to two places of decimal.

Example 1.32

Find the square root of $10\frac{2}{3}$ correct to two places of decimal.

Solution

$10\frac{2}{3} = \frac{32}{3} = 10.66\ 66\ 66$ ....

In order to find the square root correct to two places of decimal, we have to find the square root up to three places. Therefore we have to convert $\frac{2}{3}$ as a decimal correct to six places.

$$
\begin{array}{r|l}
 & 3.\ 2\ \ 6\ \ 5 \\
\hline
3 & 10.\ \overline{66}\ \overline{66}\ \overline{67} \\
 & 9 \\
\hline
62 & 1\ 66 \\
 & 1\ 24 \\
\hline
646 & 42\ 66 \\
 & 38\ 76 \\
\hline
6525 & 3\ 90\ 67 \\
 & 3\ 26\ 25 \\
\hline
 & 64\ 42
\end{array}
$$

$\sqrt{10\frac{2}{3}} \ = \ 3.265$ (approximately)

$= \ 3.27$ (correct to two places of decimal)

EXERCISE 1.6

1. Find the square root of each expression given below :

   (i) $3 \times 3 \times 4 \times 4$ (ii) $2 \times 2 \times 5 \times 5$

   (iii) $3 \times 3 \times 3 \times 3 \times 3 \times 3$ (iv) $5 \times 5 \times 11 \times 11 \times 7 \times 7$

2. Find the square root of the following :

   (i) $\frac{9}{64}$ (ii) $\frac{1}{16}$ (iii) 49 (iv) 16

3. Find the square root of each of the following by Long division method :

   (i) 2304 (ii) 4489 (iii) 3481 (iv) 529 (v) 3249

   (vi) 1369 (vii) 5776 (viii) 7921 (ix) 576 (x) 3136

4. Find the square root of the following numbers by the prime factorisation method :

   (i) 729 (ii) 400 (iii) 1764 (iv) 4096 (v) 7744

   (vi) 9604 (vii) 5929 (viii) 9216 (ix) 529 (x) 8100

5. Find the square root of the following decimal numbers :

   (i) 2.56 (ii) 7.29 (iii) 51.84 (iv) 42.25 (v) 31.36

   (vi) 0.2916 (vii) 11.56 (viii) 0.001849

6. Find the least number which must be subtracted from each of the following numbers so as to get a perfect square :

   (i) 402 (ii) 1989 (iii) 3250 (iv) 825 (v) 4000

7. Find the least number which must be added to each of the following numbers so as to get a perfect square :

   (i) 525 (ii) 1750 (iii) 252 (iv) 1825 (v) 6412

8. Find the square root of the following correct to two places of decimals :

   (i) 2 (ii) 5 (iii) 0.016 (iv) $\frac{7}{8}$ (v) $1\frac{1}{12}$

9. Find the length of the side of a square where area is 441 m^2.

10. Find the square root of the following :

    (i) $\frac{225}{3136}$ (ii) $\frac{2116}{3481}$ (iii) $\frac{529}{1764}$ (iv) $\frac{7921}{5776}$

1.7.3 Cubes

Introduction

Srinivasa Ramanujan
(1887 -1920)

This is an incident about one of the greatest mathematical geniuses S. Ramanujan. Once mathematician Prof. G.H. Hardy came to visit him in a taxi whose taxi number was 1729. While talking to Ramanujan, Hardy described that the number 1729 was a dull number. Ramanujan quickly pointed out that 1729 was indeed an interesting number. He said, it is the smallest number that can be expressed as a sum of two cubes in two different ways.

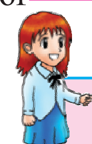
Ramanujan, an Indian Mathematician who was born in Erode contributed the theory of numbers which brought him worldwide acclamation. During his short life time, he independently compiled nearly 3900 results.

ie., $1729 = 1728 + 1 = 12^3 + 1^3$

and $1729 = 1000 + 729 = 10^3 + 9^3$

1729 is known as the Ramanujan number.

There are many other interesting patterns of cubes, cube roots and the facts related to them.



Do you know?

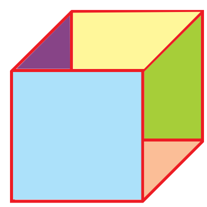
1729 is the smallest Ramanujan Number. There are an infinitely many such numbers. Few are 4104 (2, 16 ; 9, 15), 13832 (18, 20 ; 2, 24).

Cubes

We know that the word 'Cube' is used in geometry. A cube is a solid figure which has all its sides are equal.

If the side of a cube in the adjoining figure is 'a' units

then its volume is given by $a \times a \times a = a^3$ cubic units.



Here a^3 is called **"a cubed"** or **"a raised to the power three"** or **"a to the power 3"**.

Now, consider the number 1, 8, 27, 64, 125, $\cdots$

These are called **perfect cubes** or **cube numbers**.

Each of them is obtained when a number is multiplied by itself three times.

Examples: $1\times1\times1 = 1^3$, $2\times2\times2 = 2^3$, $3\times3\times3 = 3^3$, $5 \times 5 \times 5 = 5^3$

Example 1.33

Find the value of the following :

(i) 15^3 (ii) $(-4)^3$ (iii) $(1.2)^3$ (iv) $\left(\frac{-3}{4}\right)^3$

Solution

(i) $15^3 = 15\times15\times15 = 3375$

(ii) $(-4)^3 = (-4)\times(-4)\times(-4) = -64$

(iii) $(1.2)^3 = 1.2 \times 1.2 \times 1.2 = 1.728$

(iv) $\left(\frac{-3}{4}\right)^3 = \frac{(-3)\times(-3)\times(-3)}{4\times4\times4} = \frac{-27}{64}$

Observe the question (ii) Here $(-4)^3 = -64$.

Note: When a negative number is multiplied by itself an even number of times, the product is positive. But when it is multiplied by itself an odd number of times, the product is also negative. ie, $(-1)^n = \begin{cases} -1 & \text{if n is odd} \\ +1 & \text{if n is even} \end{cases}$

The following are the cubes of numbers from 1 to 20.

Numbers	Cube
1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

We are even, so are our cubes

We are odd, so are our cubes

Numbers	Cube
11	1331
12	1728
13	2197
14	2744
15	3375
16	4096
17	4913
18	5832
19	6859
20	8000

Table 2

Properties of cubes

From the above table we observe the following properties of cubes:

1. For numbers with their unit's digit as 1, their cubes also will have the unit's digit as 1.
   For example: $1^3 = 1$; $11^3 = 1331$; $21^3 = 9261$; $31^3 = 29791$.
2. The cubes of the numbers with 1, 4, 5, 6, 9 and 0 as unit digits will have the same unit digits.
   For example: $14^3 = 2744$; $15^3 = 3375$; $16^3 = 4096$; $20^3 = 8000$.
3. The cube of numbers ending in unit digit 2 will have a unit digit 8 and the cube of the numbers ending in unit digit 8 will have a unit digit 2.
   For example: $(12)^3 = 1728$; $(18)^3 = 5832$.
4. The cube of the numbers with unit digits as 3 will have a unit digit 7 and the cube of numbers with unit digit 7 will have a unit digit 3.
   For example: $(13)^3 = 2197$; $(27)^3 = 19683$.
5. The cubes of even numbers are all even; and the cubes of odd numbers are all odd.

Adding consecutive odd numbers

Observe the following pattern of sums of odd numbers.

$$1 = 1 = 1^3$$

Next 2 odd numbers, $3 + 5 = 8 = 2^3$

Next 3 odd numbers, $7 + 9 + 11 = 27 = 3^3$

Next 4 odd numbers, $13 + 15 + 17 + 19 = 64 = 4^3$

Next 5 odd numbers, $21 + 23 + 25 + 27 + 29 = 125 = 5^3$

Is it not interesting?

Example 1.34

Is 64 a perfect cube?

Solution

$$64 = \underbrace{2\times2\times2}\times\underbrace{2\times2\times2}$$

$$= 2^3\times2^3 = (2\times2)^3 = 4^3$$

$\therefore$ 64 is a perfect cube.

Prime factorization

$$\begin{array}{r|l} 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

Example 1.35

Is 500 a perfect cube?

Solution

$$500 = 2\times2\times\underbrace{5\times5\times5}$$

There are three 5's in the product but only two 2's.

So 500 is not a perfect cube.

Prime factorization

$$\begin{array}{r|l} 2 & 500 \\ \hline 2 & 250 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

Example 1.36

Is 243 a perfect cube? If not find the smallest number by which 243 must be multiplied to get a perfect cube.

Prime factorization

$$\begin{array}{r|l} 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Solution

$$243 = \underbrace{3\times3\times3}\times3\times3$$

In the above factorisation, 3×3 remains after grouping the $3's$ in triplets. $\therefore$ 243 is not a perfect cube.

To make it a perfect cube we multiply it by 3.

$$243\times3 = \underbrace{3\times3\times3}\times\underbrace{3\times3\times3}$$

$$729 = 3^3\times3^3 = (3\times3)^3$$

$$729 = 9^3 \text{ which is a perfect cube.}$$

$\therefore$ 729 is a perfect cube.

Prime factorization

$$\begin{array}{r|l} 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

MATHEMATICS

1.7.4 Cube roots

If the volume of a cube is 125 cm^3, what would be the length of its side? To get the length of the side of the cube, we need to know a number whose cube is 125. To find the cube root, we apply inverse operation in finding cube.

Symbol

$\sqrt[3]{\ }$ denotes "cube - root"

For example:

We know that $2^3 = 8$, the cube root of 8 is 2.

We write it mathematically as

$$\sqrt[3]{8} = (8)^{1/3} = (2^3)^{1/3} = 2^{3/3} = 2$$

Some more examples:

(i) $\sqrt[3]{125} = \sqrt[3]{5^3} = (5^3)^{1/3} = 5^{3/3} = 5^1 = 5$

(ii) $\sqrt[3]{64} = \sqrt[3]{4^3} = (4^3)^{1/3} = 4^{3/3} = 4^1 = 4$

(iii) $\sqrt[3]{1000} = \sqrt[3]{10^3} = (10^3)^{1/3} = 10^{3/3} = 10^1 = 10$

Cube root through prime factorization method

Method of finding the cube root of a number

Step 1 : Resolve the given number into prime factors.

Step 2 : Write these factors in triplets such that all three factors in each triplet are equal.

Step 3 : From the product of all factors, take one from each triplet that gives the cube root of a number.

Example 1.37

Find the cube root of 512.

Solution

$$\sqrt[3]{512} = (512)^{\frac{1}{3}}$$
$$= ((2\times2\times2)\times(2\times2\times2)\times(2\times2\times2))^{\frac{1}{3}}$$
$$= (2^3\times2^3\times2^3)^{\frac{1}{3}}$$
$$= \left(2^9\right)^{\frac{1}{3}} = 2^3$$
$$\sqrt[3]{512} = 8.$$

Prime factorization

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Example 1.38

Find the cube root of 27×64

Solution

Resolving 27 and 64 into prime factors, we get

$$\sqrt[3]{27} = (3\times3\times3)^{\frac{1}{3}} = (3^3)^{\frac{1}{3}}$$

Prime factorization

3	27
3	9
3	3
	1

$$\sqrt[3]{27} = 3$$

$$\sqrt[3]{64} = (2\times2\times2\times2\times2\times2)^{\frac{1}{3}}$$

$$= (2^6)^{\frac{1}{3}} = 2^2 = 4$$

$$\sqrt[3]{64} = 4$$

$$\sqrt[3]{27\times64} = \sqrt[3]{27}\times\sqrt[3]{64}$$

$$= 3\times4$$

$$\sqrt[3]{27\times64} = 12$$

Prime factorization

2	64
2	32
2	16
2	8
2	4
2	2
	1

Example 1.39

Is 250 a perfect cube? If not, then by which smallest natural number should 250 be divided so that the quotient is a perfect cube?

Solution

$$250 = 2\times\underbrace{5\times5\times5}$$

The prime factor 2 does not appear in triplet. Therefore 250 is not a perfect cube.

Prime factorization

2	250
5	125
5	25
5	5
	1

Since in the factorisation, 2 appears only one time. If we divide the number 250 by 2, then the quotient will not contain 2. Rest can be expressed in cubes.

$$\therefore 250 \div 2 = 125$$

$$= 5\times5\times5 = 5^3.$$

$\therefore$ The smallest number by which 250 should be divided to make it a perfect cube is 2.

Cube root of a fraction

$$\boxed{\text{Cube root of a fraction} = \frac{\text{Cube root of its numerator}}{\text{Cube root of its denominator}}}$$

(i.e.) $$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}} = \left(\frac{a}{b}\right)^{\frac{1}{3}} = \frac{(a)^{\frac{1}{3}}}{(b)^{\frac{1}{3}}}$$

Example 1.40

Find the cube root of $\frac{125}{216}$.

Solution

Resolving 125 and 216 into prime factors, we get

$$125 = \underbrace{5\times5\times5}$$

Prime factorization

5	125
5	25
5	5
	1

$$\sqrt[3]{125} = 5$$

$$216 = \underbrace{2 \times 2 \times 2}\times\underbrace{3 \times 3 \times 3}$$

$$\therefore \sqrt[3]{216} = 2 \times 3$$

$$\therefore \sqrt[3]{216} = 6$$

$$\therefore \sqrt[3]{\frac{125}{216}} = \frac{5}{6}.$$

Prime factorization

2	216
2	108
2	54
3	27
3	9
3	3
	1

Example 1.41

Find the cube root of $\frac{-512}{1000}$

Solution

$$-512 = \underbrace{-8 \times -8 \times -8}$$

$$\sqrt[3]{-512} = -8$$

$$1000 = 5 \times 5 \times 5 \times 2 \times 2 \times 2$$

$$\sqrt[3]{1000} = 10$$

$$\sqrt[3]{\frac{-512}{1000}} = \frac{-8}{10}$$

$$\sqrt[3]{\frac{-512}{1000}} = \frac{-4}{5}$$

Prime factorization

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factorization

5	1000
5	200
5	40
2	8
2	4
2	2
	1

Think it!

$$\sqrt[3]{-x^3} = \sqrt[3]{(-x)\times(-x)\times(-x)}$$

$$= -x.$$

The cube root of a negative number is negative.

Example 1.42

Find the cube root of 0.027

Solution

$$\sqrt[3]{0.027} = \sqrt[3]{\frac{27}{1000}}$$

$$= \sqrt[3]{\frac{3 \times 3 \times 3}{10 \times 10 \times 10}}$$

$$= \frac{\sqrt[3]{3^3}}{\sqrt[3]{10^3}} = \frac{3}{10}$$

$$\sqrt[3]{0.027} = 0.3$$

Example 1.43

Evaluate $\frac{\sqrt[3]{729} - \sqrt[3]{27}}{\sqrt[3]{512} + \sqrt[3]{343}}$

Solution

$$\sqrt[3]{729} = \sqrt[3]{9^3} = 9$$

$$\sqrt[3]{27} = \sqrt[3]{3^3} = 3$$

Prime factorization

3	27
3	9
3	3
	1

Prime factorization

7	343
7	49
7	7
	1

$$\sqrt[3]{512} = \sqrt[3]{8^3} = 8$$

$$\sqrt[3]{343} = \sqrt[3]{7^3} = 7$$

$$\therefore \frac{\sqrt[3]{729} - \sqrt[3]{27}}{\sqrt[3]{512} + \sqrt[3]{343}} = \frac{9-3}{8+7}$$

$$= \frac{6}{15} = \frac{2}{5}$$

Prime factorization

$$\begin{array}{r|l} 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Prime factorization

$$\begin{array}{r|l} 2 & 512 \\ \hline 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline 2 & 32 \\ \hline 2 & 16 \\ \hline 2 & 8 \\ \hline 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array}$$

EXERCISE 1.7

1. Choose the correct answer for the following :

(i) Which of the following numbers is a perfect cube?

(A) 125 (B) 36 CC) 75 (D) 100

(ii) Which of the following numbers is not a perfect cube?

(A) 1331 (B) 512 (C) 343 (D) 100

(iii) The cube of an odd natural number is

(A) Even (B) Odd

(C) May be even, May be odd (D) Prime number

(iv) The number of zeros of the cube root of 1000 is

(A) 1 (B) 2 (C) 3 (D) 4

(v) The unit digit of the cube of the number 50 is

(A) 1 (B) 0 (C) 5 (D) 4

(vi) The number of zeros at the end of the cube of 100 is

(A) 1 (B) 2 (C) 4 (D) 6

(vii) Find the smallest number by which the number 108 must be multiplied to obtain a perfect cube

(A) 2 (B) 3 (C) 4 (D) 5

(viii) Find the smallest number by which the number 88 must be divided to obtain a perfect cube

(A) 11 (B) 5 (C) 7 (D) 9

(ix) The volume of a cube is 64 cm^3. The side of the cube is

(A) 4 cm (B) 8 cm (C) 16 cm (D) 6 cm

(x) Which of the following is false?

(A) Cube of any odd number is odd.

(B) A perfect cube does not end with two zeros.

(C) The cube of a single digit number may be a single digit number.
(D) There is no perfect cube which ends with 8.

2. Check whether the following are perfect cubes?

(i) 400 (ii) 216 (iii) 729 (iv) 250
(v) 1000 (vi) 900

3. Which of the following numbers are not perfect cubes?

(i) 128 (ii) 100 (iii) 64 (iv) 125
(v) 72 (vi) 625

4. Find the smallest number by which each of the following number must be divided to obtain a perfect cube.

(i) 81 (ii) 128 (iii) 135 (iv) 192
(v) 704 (vi) 625

5. Find the smallest number by which each of the following number must be multiplied to obtain a perfect cube.

(i) 243 (ii) 256 (iii) 72 (iv) 675
(v) 100

6. Find the cube root of each of the following numbers by prime factorisation method:

(i) 729 (ii) 343 (iii) 512 (iv) 0.064
(v) 0.216 (vi) $5\frac{23}{64}$ (vii) – 1.331 (viii) – 27000

7. The volume of a cubical box is 19.683 cu. cm. Find the length of each side of the box.

1.8 Approximation of Numbers

In our daily life we need to know approximate values or measurements.

Benjamin bought a Lap Top for ₹ 59,876. When he wants to convey this amount to others, he simply says that he has bought it for ₹ 60,000. This is the **approximate value** which is given in thousands only.

Vasanth buys a pair of chappals for ₹ 599.95. This amount may be considered approximately as ₹ 600 for convenience.

A photo frame has the dimensions of 35.23 cm long and 25.91 cm wide. If we want to check the measurements with our ordinary scale, we cannot measure accurately because our ordinary scale is marked in tenths of centimetre only.

In such cases, we can check the length of the photo frame 35.2 cm to the nearest tenth or 35 cm to the nearest integer value.

In the above situations we have taken the approximate values for our convenience. This type of considering the nearest value is called **'Rounding off'** the digits. Thus the approximate value corrected to the required number of digits is known as 'Rounding off' the digits.

Sometimes it is possible only to give approximate value, because

(a) If we want to say the population of a city, we will be expressing only in approximate value say 30 lakhs or 25 lakhs and so on.

(b) When we say the distance between two cities, we express in round number 350 km not 352.15 kilometres.

While rounding off the numbers we adopt the following principles.

(i) If the number next to the desired place of correction is less than 5, give the answer up to the desired place as it is.

(ii) If the number next to the desired place of correction is 5 and greater than 5 add 1 to the number in the desired place of correction and give the answer.

The symbol for approximation is usually denoted by $\simeq$.

Activity

Take an A4 sheet. Measure its length and breadth. How do you express it in cm's approximately.

Let us consider some examples to find the approximate values of a given number.

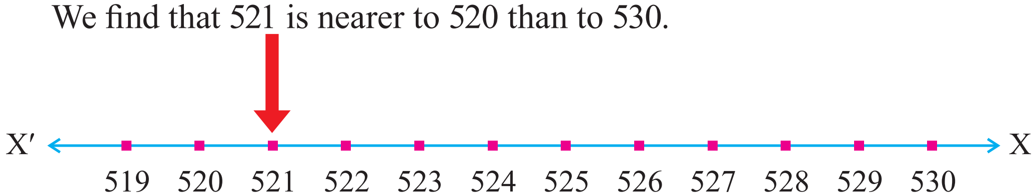
Take the number 521.

Approximation nearest to TEN

Illustration

Consider multiples of 10 before and after 521. (i.e. 520 and 530)

We find that 521 is nearer to 520 than to 530.

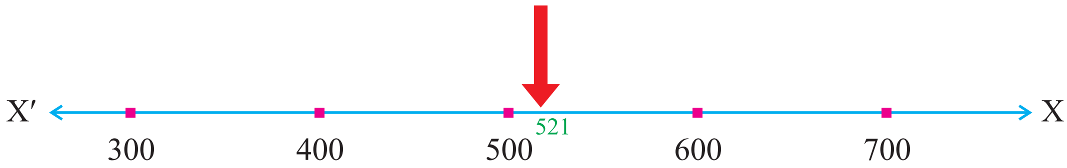


$\therefore$ The approximate value of 521 is 520 in this case.

Approximation nearest to HUNDRED

Illustration

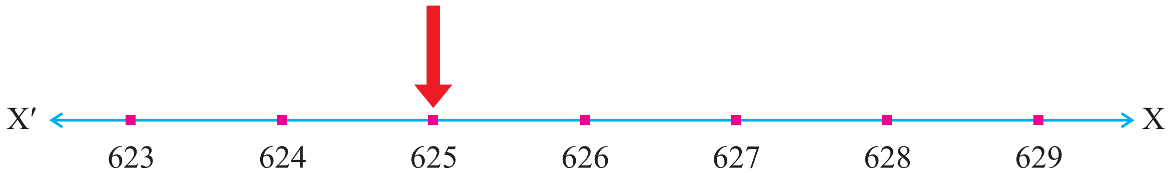
(i) Consider multiples of 100 before and after 521. (i.e. 500 and 600)



We find that 521 is nearer to 500 than to 600. So, in this case, the approximate value of 521 is 500.

(ii) Consider the number 625

Suppose we take the number line, unit by unit.



In this case, we cannot say whether 625 is nearer to 624 or 626 because it is exactly midway between 624 and 626. However, by convention we say that it is nearer to 626 and hence its approximate value is taken to be 626.

Suppose we consider multiples of 100, then 625 will be approximated to 600 and not 700.

Some more examples

For the number 47,618

(a) Approximate value correct to the nearest tens = 47,620

(b) Approximate value correct to the nearest hundred = 47,600

(c) Approximate value correct to the nearest thousand = 48,000

(d) Approximate value correct to the nearest ten thousand = 50,000

Decimal Approximation

Illustration

Consider the decimal number 36.729

(a) It is 36.73 correct to two decimal places. (Since the last digit 9>5, we add 1 to 2 and make it 3).

$\therefore$ 36.729 $\simeq$ 36.73 (Correct to two decimal places)

(b) Look at the second decimal in 36.729, Here it is 2 which is less than 5, so we leave 7 as it is. $\therefore$ 36.729 $\simeq$ 36.7 (Correct to one decimal place)

Illustration

Consider the decimal number 36.745

(a) It's approximation is 36.75 correct to two decimal places. Since the last digit is 5, We add 1 to 4 and make it 5.

(b) It's approximation is 36.7 correct to one decimal place. Since the second decimal is 4, which is less than 5, we leave 7 as it is.

$\therefore$ 36.745 $\simeq$ 36.7

Ravi has the following numbered cards

2	3	1	5	9

Help him to find the approximate value correct to the nearest 20,000.

Find the greatest number using the method of approximation

a. $201120112011 + \frac{7}{18}$

b. $201120112011 - \frac{7}{18}$

c. $201120112011 \times \frac{7}{18}$

d. $201120112011 \div \frac{7}{18}$

Illustration

Consider the decimal number 2.14829

(i) Approximate value correct to one decimal place is 2.1

(ii) Approximate value correct to two decimal place is 2.15

(iii) Approximate value correct to three decimal place is 2.148

(iv) Approximate value correct to four decimal place is 2.1483

Example 1.44

Round off the following numbers to the nearest integer:

(a) 288.29 (b) 3998.37 (c) 4856.795 (d) 4999.96

Solution

(a) 288.29 $\simeq$ 288 (b) 3998.37 $\simeq$ 3998

(Here, the tenth place in the above numbers are less than 5. Therefore all the integers are left as they are.)

(c) 4856.795 $\simeq$ 4857 (d) 4999.96 $\simeq$ 5000

[Here, the tenth place in the above numbers are greater than 5. Therefore the integer values are increased by 1 in each case.]

EXERCISE 1.8

1. Express the following correct to two decimal places:

(i) 12.568 (ii) 25.416 kg (iii) 39.927 m

(iv) 56.596 m (v) 41.056 m (vi) 729.943 km

2. Express the following correct to three decimal places:

(i) 0.0518 m (ii) 3.5327 km

(iii) 58.2936l (iv) 0.1327 gm

(v) 365.3006 (vi) 100.1234

3. Write the approximate value of the following numbers to the accuracy stated:

   (i) 247 to the nearest ten. (ii) 152 to the nearest ten.

   (iii) 6848 to the nearest hundred. (iv) 14276 to the nearest ten thousand.

   (v) 3576274 to the nearest Lakhs. (vi) 104, 3567809 to the nearest crore

4. Round off the following numbers to the nearest integer:

   (i) 22.266 (ii) 777.43 (iii) 402.06

   (iv) 305.85 (v) 299.77 (vi) 9999.9567

1.9. Playing with Numbers

Mathematics is a subject with full of fun, magic and wonders. In this unit, we are going to enjoy with some of this fun and wonder.

(a) Numbers in General form

Let us take the number 42 and write it as

$$42 = 40 + 2 = 10 \times 4 + 2$$

Similarly, the number 27 can be written as

$$27 = 20 + 7 = 10 \times 2 + 7$$

In general, any two digit number ***ab*** made of digits ***'a'*** and ***'b'*** can be written as

$$ab = 10 \times a + b = 10\,a + b$$

$$ba = 10 \times b + a = 10\,b + a$$

Here *ab* does not mean $a \times b$ but digits.

Now let us consider the number 351.

This is a three digit number. It can also be written as

$$351 = 300 + 50 + 1 = 100 \times 3 + 10 \times 5 + 1 \times 1$$

In general, a 3-digit number *abc* made up of digit *a*, *b* and *c* is written as

$$abc = 100 \times a + 10 \times b + 1 \times c$$

$$= 100a + 10b + 1c$$

In the same way, the three digit numbers *cab* and *bca* can be written as

$$cab = 100c + 10a + b$$

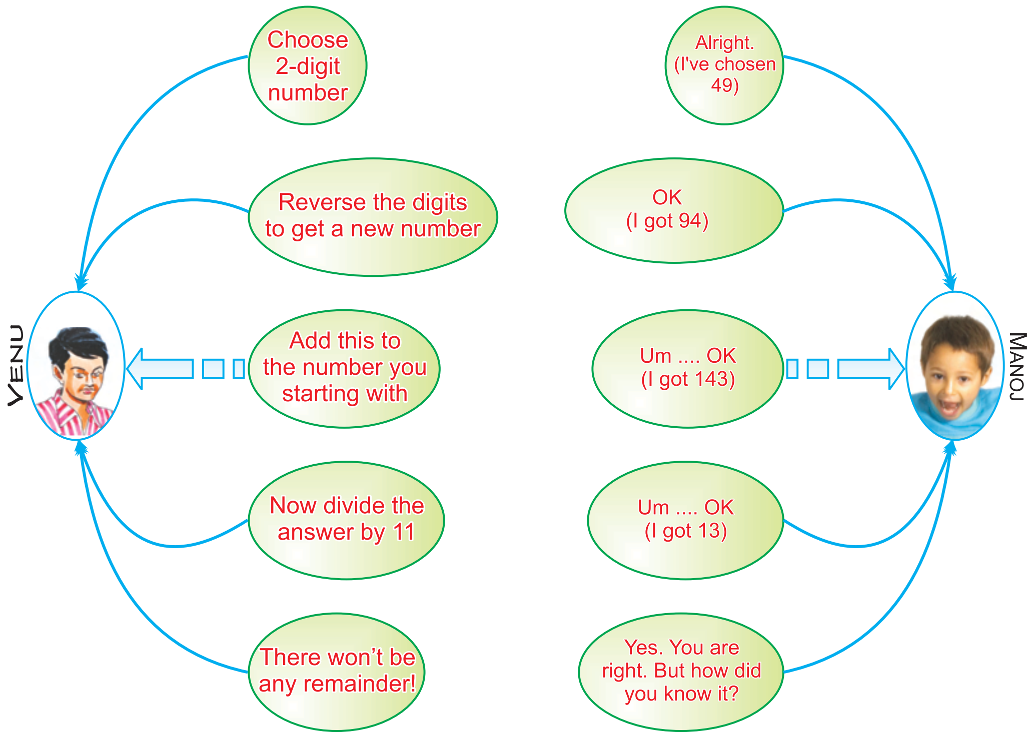
$$bca = 100b + 10c + a$$

(b) Games with Numbers

(i) Reversing the digits of a two digit number

Venu asks Manoj to think of a 2 digit number, and then to do whatever he asks him to do, to that number. Their conversation is shown in the following figure. Study the figure carefully before reading on.

Conversation between Venu and Manoj:



Now let us see if we can explain Venu's "trick". Suppose, Manoj chooses the number ab, which is a short form for the 2 -digit number $10a + b$. On reversing the digits, he gets the number $ba = 10b + a$. When he adds the two numbers he gets :

$$\begin{aligned}(10a + b) + (10b + a) &= 11a + 11b \\ &= 11(a + b)\end{aligned}$$

So the sum is always a multiple of 11, just as Venu had claimed.

Dividing the answer by 11, we get $(a + b)$

(i.e.) Simply adding the two digit number.

(c) Identify the pattern and find the next three terms

Study the pattern in the sequence.

(i) 3, 9, 15, 21, (Each term is 6 more than the term before it)

If this pattern continues, then the next terms are ___ , ___ and ___ .

(ii) 100, 96, 92, 88, ___ , ___ , ___ . (Each term is 4 less than the previous term)

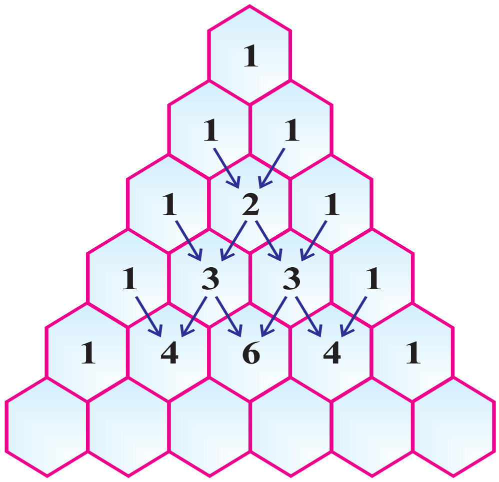
(iii) 7, 14, 21, 28, ___ , ___ , ___ . (Multiples of 7)

(iv) 1000, 500, 250, ___ , ___,___. (Each term is half of the previous term)

(v) 1, 4, 9, 16, ___ , ___ , ___ . (Squares of the Natural numbers)

(d) Number patterns in Pascal's Triangle

The triangular shaped, pattern of numbers given below is called **Pascal's Triangle**.



Activity

Identify the number pattern in **Pascal's triangle** and complete the 6^{th} row.

3 × 3 Magic Square

6	11	10
13	9	5
8	7	12

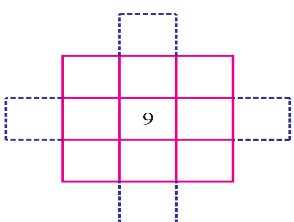
Look at the above table of numbers. This is called a 3×3 magic square. In a magic square, the sum of the numbers in each row, each column, and along each diagonal is the same.

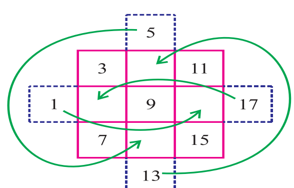
In this magic square, the magic sum is 27. Look at the middle number. The magic sum is 3 times the middle number. Once 9 is filled in the centre, there are eight boxes to be filled. Four of them will be below 9 and four of them above it. They could be,

(a) $5, 6, 7, 8$ and $10, 11, 12, 13$ with a difference of 1 between each number.

(b) $1, 3, 5, 7$ and $11, 13, 15, 17$ with a difference of 2 between them or it can be any set of numbers with equal differences such as $-11, -6, -1, 4$ and 14, 19, 24, 29 with a difference of 5.

Once we have decided on the set of numbers, say 1, 3, 5, 7 and $11, 13, 15, 17$ draw four projections out side the square, as shown in below figure and enter the numbers in order, as shown in a diagonal pattern.The number from each of the projected box is transferred to the empty box on the opposite side.





3	13	11
17	9	1
7	5	15

Activity

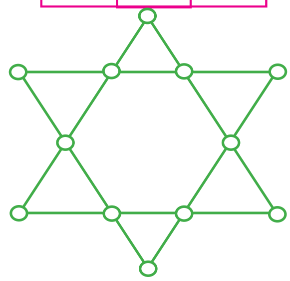
MAGIC SQUARE

Murugan has 9 pearls each of worth 1 to 9 gold coins. Could you help him to distribute them among his three daughters equally.

8		6
	5	
		2

MAGIC STAR

In the adjacent figure, use the numbers from 1 to 12 to fill up the circles within the star such that the sum of each line is 26. A number can be used twice atmost.

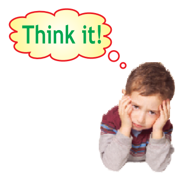


SU DO KU

Use all the digits 1, 2, ..., 9 to fill up each rows, columns and squares of different colours inside without repetition.

3	1	2		9	5		7	6
5		9	1		7		8	2
4		7	2	6	3	5		
9			7			2	4	
	2	8		1			9	3
	3		9	8	2		5	7
	4	5	6				3	1
1	7		3	5	8	9		4
8		3	4	2		7		5

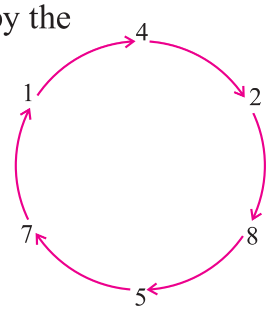
A three digit register number of a car is a square number. The reverse of this number is the register number of another car which is also a square number. Can you give the possible register numbers of both cars?



The Revolving Number

1 4 2 8 5 7

First set out the digits in a circle. Now multiply 142857 by the number from 1 to 6.



$$\begin{array}{r}142857\\ \times 1\\ \hline 142857\\ \hline\end{array}\qquad\begin{array}{r}142857\\ \times 2\\ \hline 285714\\ \hline\end{array}\qquad\begin{array}{r}142857\\ \times 3\\ \hline 428571\\ \hline\end{array}$$

$$\begin{array}{r}142857\\ \times 4\\ \hline 571428\\ \hline\end{array}\qquad\begin{array}{r}142857\\ \times 5\\ \hline 714285\\ \hline\end{array}\qquad\begin{array}{r}142857\\ \times 6\\ \hline 857142\\ \hline\end{array}$$

We observe that the number starts revolving the same digits in different combinations. These numbers are arrived at starting from a different point on the circle.

EXERCISE 1.9

1. Complete the following patterns:

   (i) 40, 35, 30, ___ , ___ , ___ .

   (ii) 0, 2, 4, ___ , ___ , ___ .

   (iii) 84, 77, 70, ___ , ___ , ___ .

   (iv) 4.4, 5.5, 6.6, ___ , ___ , ___ .

   (v) 1, 3, 6, 10, ___ , ___ , ___ .

   (vi) 1, 1, 2, 3, 5, 8, 13, 21, ___ , ___ , ___ .

   (This sequence is called FIBONACCI SEQUENCE)

   (vii) 1, 8, 27, 64, ___ , ___ , ___ .

Activity

PUZZLE

- Choose a number
- Add 9 to it
- Double the answer
- Add 3 with the result
- Multiply the result by 3
- Subtract 3 from it
- Divide it by 6
- Subtract the number that you have chosen first from the answer.
- What is your answer?

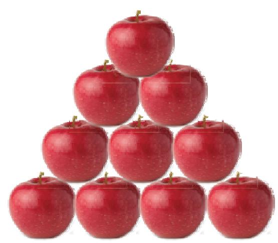
ANSWER : Ten

2. A water tank has steps inside it. A monkey is sitting on the top most step. (ie, the first step) The water level is at the ninth step.

   (a) He jumps 3 steps down and then jumps back 2 steps up. In how many jumps will he reach the water level ?

   (b) After drinking water, he wants to go back. For this, he jumps 4 steps up and then jumps back 2 steps down in every move. In how many jumps will he reach back the top step ?

3. A vendor arranged his apples as in the following pattern :



   (a) If there are ten rows of apples, can you find the total number of apples without actually counting?

   (b) If there are twenty rows, how many apples will be there in all?

   Can you recognize a pattern for the total number of apples? Fill this chart and try!

Rows	1	2	3	4	5	6	7	8	9
Total apples	1	3	6	10	15				

Concept Summary

- Rational numbers are closed under the operations of addition, subtraction and multiplication.
- The collection of non-zero rational numbers is closed under division.
- The operations addition and multiplication are commutative and associative for rational numbers.
- 0 is the additive identity for rational numbers.
- 1 is the multplicative identity for rational numbers.
- Multiplication of rational numbers is distributive over addition and subtraction.
- The additive inverse of $\frac{a}{b}$ is $\frac{-a}{b}$ and vice-versa.
- The reciprocal or multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$.
- Between two rational numbers, there are countless rational numbers.
- The seven laws of exponents are :

  If a and b are real numbers and m, n are whole numbers then

  (i) $a^m \times a^n = a^{m+n}$

  (ii) $a^m \div a^n = a^{m-n}$, where $a \neq 0$

  (iii) $a^0 = 1$, where $a \neq 0$

  (iv) $a^{-m} = \frac{1}{a^m}$, where $a \neq 0$

  (v) $(a^m)^n = a^{mn}$

  (vi) $a^m \times b^m = (ab)^m$

  (vii) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$ where $b \neq o$
- Estimated value of a number equidistant from the other numbers is always greater than the given number and nearer to it.

2 Measurements

2.1 Introduction

Measuring is a skill. It is required for every individual in his / her life. Everyone of us has to measure something or the other in our daily life. For instance, we measure

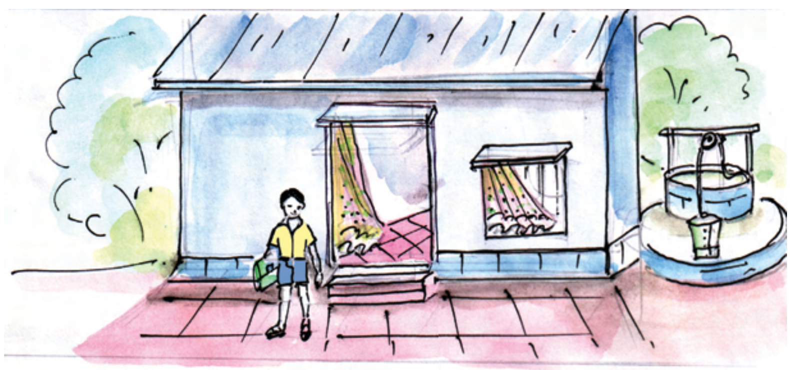


Fig. 2.1

(i) the length of a rope required for drawing water from a well,

(ii) the length of the curtain cloth required for our doors and windows,

(iii) the size of the floor in a room to be tiled in our house and

(iv) the length of cloth required for school uniform dress.

In all the above situations, the idea of 'measurements' comes in.

The branch of mathematics which deals with the measure of lengths, angles, areas, perimeters in plane figures and surface areas, volumes in solid figures is called '**measurement and mensuration**'.

Recall

Let us recall the following definitions which we have learnt in class VII.

(i) Area

Area is the portion inside the closed figure in a plane surface.

The word '**peri**' in Greek means '**around**' and '**meter**' means '**measure**'.

(ii) Perimeter

The perimeter of a closed figure is the total measure of the boundary.

Thus, the perimeter means measuring around a figure or measuring along a curve.

Can you identify the shape of the following objects?

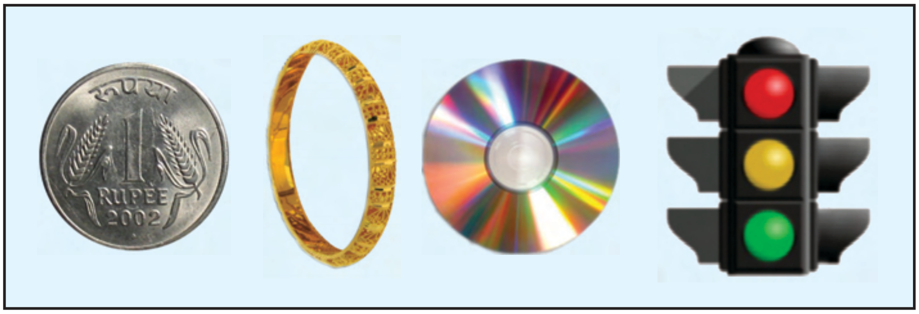


Fig. 2.2

The shape of each of these objects is a 'circle'.

(iii) Circle

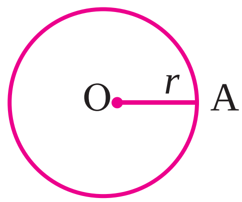


Fig. 2.3

Let 'O' be the centre of a circle with radius 'r' units ($\overline{OA}$).

Area of a circle, $A = \pi r^2$ sq.units.

Perimeter or circumference of a circle,

$$P = 2\pi r \text{ units,}$$

where $\pi \simeq \frac{22}{7}$ or 3.14.

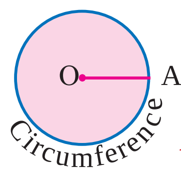


Fig. 2.4

Note: The central angle of a circle is 360°.

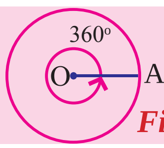


Fig. 2.5

Activity

Take a cardboard and draw circles of different radii. Cut the circles and find their areas and perimeters.

S. No.	Radius	Area	Perimeter
1.			
2.			
3.			

2.2 Semi circles and Quadrants

2.2.1 Semicircle

Have you ever noticed the sky during night time after 7 days of new moon day or full moon day?

What will be the shape of the moon?

It looks like the shape of Fig. 2.6.

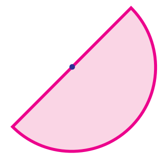


Fig. 2.6

How do you call this?

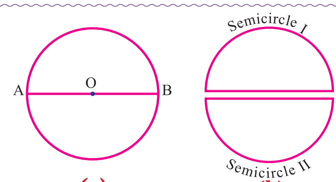
This is called a semicircle. [Half part of a circle]

The two equal parts of a circle divided by its diameter are called semicircles.

Activity

How will you get a semicircle from a circle?

Take a cardboard of circular shape and cut it through its diameter $\overline{AB}$.



(a) *Fig. 2.7* (b)

Note: The central angle of the semicircle is 180°.

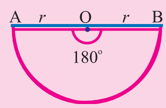


Fig. 2.8

(a) Perimeter of a semicircle

$$\text{Perimeter, P} = \frac{1}{2} \times (\text{circumference of a circle}) + 2 \times r \text{ units}$$

$$= \frac{1}{2} \times 2\pi r + 2r$$

$$\text{P} = \pi r + 2r = (\pi + 2)\, r \text{ units}$$

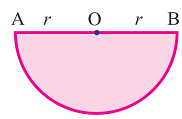


Fig. 2.9

(b) Area of a semicircle

$$\text{Area, A} = \frac{1}{2} \times (\text{Area of a circle})$$

$$= \frac{1}{2} \times \pi r^2$$

$$\text{A} = \frac{\pi r^2}{2} \text{ sq. units.}$$

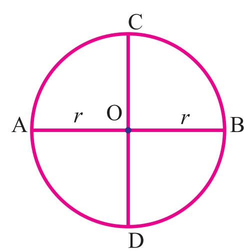


Fig. 2.10

4.2.2 Quadrant of a circle

Cut the circle through two of its perpendicular diameters. We get four equal parts of the circle. Each part is called a quadrant of the circle. We get four quadrants OCA, OAD, ODB and OBC while cutting the circle as shown in the Fig. 2.11.

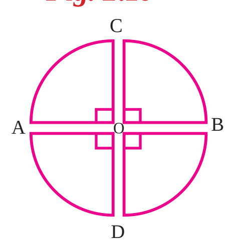


Fig. 2.11

Note: The central angle of the quadrant is 90°.

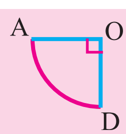


Fig. 2.12

(a) Perimeter of a quadrant

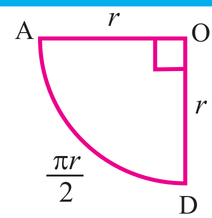


Fig. 2.13

$$\text{Perimeter, P} = \frac{1}{4} \times (\text{circumference of a circle}) + 2r \text{ units}$$
$$= \frac{1}{4} \times 2\pi r + 2r$$
$$\text{P} = \frac{\pi r}{2} + 2r = \left(\frac{\pi}{2} + 2\right) r \text{ units}$$

(b) Area of a quadrant

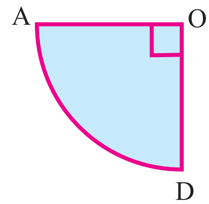


Fig. 2.14

$$\text{Area, A} = \frac{1}{4} \times (\text{Area of a circle})$$
$$\text{A} = \frac{1}{4} \times \pi r^2 \text{ sq.units}$$

Example 2.1

Find the perimeter and area of a semicircle whose radius is 14 cm.

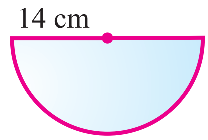


Fig. 2.15

Solution

Given: Radius of a semicircle, $r = 14$ cm

$$\text{Perimeter of a semicircle, P} = (\pi + 2)\, r \text{ units}$$
$$\therefore \text{P} = \left(\frac{22}{7} + 2\right) \times 14$$
$$= \left(\frac{22 + 14}{7}\right) \times 14 = \frac{36}{7} \times 14 = 72$$

Perimeter of the semicircle $= 72$ cm.

$$\text{Area of a semicircle, A} = \frac{\pi r^2}{2} \text{ sq. units}$$
$$\therefore \text{A} = \frac{22}{7} \times \frac{14 \times 14}{2} = 308 \text{ cm}^2.$$

Example 2.2

The radius of a circle is 21 cm. Find the perimeter and area of a quadrant of the circle.

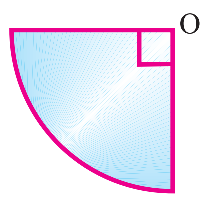


Fig. 2.16

Solution

Given: Radius of a circle, $r = 21$ cm

$$\text{Perimeter of a quadrant, P} = \left(\frac{\pi}{2} + 2\right) r \text{ units}$$
$$= \left(\frac{22}{7 \times 2} + 2\right) \times 21 = \left(\frac{22}{14} + 2\right) \times 21$$
$$\text{P} = \left(\frac{22 + 28}{14}\right) \times 21 = \frac{50}{14} \times 21$$
$$= 75 \text{ cm.}$$

$$\text{Area of a quadrant, A} = \frac{\pi r^2}{4} \text{ sq. units}$$
$$\text{A} = \frac{22}{7} \times \frac{21 \times 21}{4}$$
$$= 346.5 \text{ cm}^2.$$

Example 2.3

The diameter of a semicircular grass plot is 14 m. Find the cost of fencing the plot at ₹ 10 per metre .

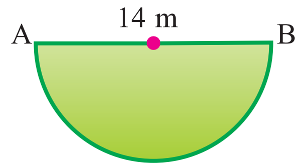


Fig. 2.17

Solution

Given: Diameter, d = 14 m.

$\therefore$ Radius of the plot, $r = \frac{14}{2} = 7$ m.

To fence the semicircular plot, we have to find the perimeter of it.

$$\text{Perimeter of a semicircle, P} = (\pi + 2) \times \text{r units}$$
$$= \left(\frac{22}{7} + 2\right) \times 7$$
$$= \left(\frac{22 + 14}{7}\right) \times 7$$
$$\text{P} = 36 \text{ m}$$

Cost of fencing the plot for 1 metre = ₹ 10

$\therefore$ Cost of fencing the plot for 36 metres = 36 × 10 = ₹ 360.

Example 2.4

The length of a chain used as the boundary of a semicircular park is 36 m. Find the area of the park.

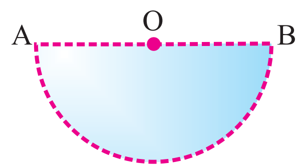


Fig. 2.18

Solution

Given:

Length of the boundary = Perimeter of a semicircle

$$\therefore (\pi + 2)r = 36 \text{ m} = \left(\frac{22}{7} + 2\right) \times r = 36$$
$$\left(\frac{22 + 14}{7}\right) \times r = 36 \text{ m} = \frac{36}{7} \times r = 36 \Rightarrow r = 7 \text{ m}$$

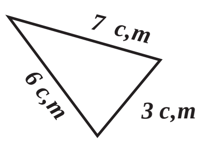
Area of the park = Area of the semicircle

$$\text{A} = \frac{\pi r^2}{2} \text{ sq. units} = \frac{22}{7} \times \frac{7 \times 7}{2} = 77 \text{ m}^2$$

$\therefore$ Area of the park = 77 m^2.

Activity

A rod is bent in the shape of a triangle as shown in the figure. Find the length of the side if it is bent in the shape of a square?



EXERCISE 2.1

1. Choose the correct answer:

(i) Area of a semicircle is _________ times the area of the circle.

(A) two (B) four (C) one-half (D) one-quarter

(ii) Perimeter of a semicircle is ________

(A) $\left(\frac{\pi+2}{2}\right)$ r units (B) $(\pi+2)$ r units

(C) 2r units (D) $(\pi+4)$r units

(iii) If the radius of a circle is 7 m, then the area of the semicircle is _______

(A) 77 m^2 (B) 44 m^2 (C) 88 m^2 (D) 154 m^2

(iv) If the area of a circle is 144 cm^2, then the area of its quadrant is _______

(A) 144 cm^2 (B) 12 cm^2 (C) 72 cm^2 (D) 36 cm^2

(v) The perimeter of the quadrant of a circle of diameter 84 cm is _______

(A) 150 cm (B) 120 cm (C) 21 cm (D) 42 cm

(vi) The number of quadrants in a circle is _______

(A) 1 (B) 2 (C) 3 (D) 4

(vii) Quadrant of a circle is ______ of the circle.

(A) one-half (B) one-fourth (C) one-third (D) two-thirds

(viii) The central angle of a semicircle is _________

(A) 90° (B) 270° (C) 180° (D) 360°

(ix) The central angle of a quadrant is _______

(A) 90° (B) 180° (C) 270° (D) 0°

(x) If the area of a semicircle is 84 cm^2, then the area of the circle is _______

(A) 144 cm^2 (B) 42 cm^2 (C) 168 cm^2 (D) 288 cm^2

2. Find the perimeter and area of semicircles whose radii are,

(i) 35 cm (ii) 10.5 cm (iii) 6.3 m (iv) 4.9 m

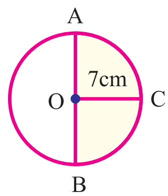
3. Find the perimeter and area of semicircles whose diameters are,

(i) 2.8 cm (ii) 56 cm (iii) 84 cm (iv) 112 m

4. Calculate the perimeter and area of a quadrant of the circles whose radii are,

(i) 98 cm (ii) 70 cm (iii) 42 m (iv) 28 m

5. Find the area of the semicircle ACB and the quadrant BOC in the given figure.



6. A park is in the shape of a semicircle with radius 21 m. Find the cost of fencing it at the cost of ₹ 5 per metre.

2.3 Combined Figures

Fig. 2.19

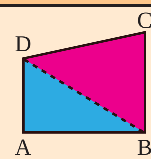
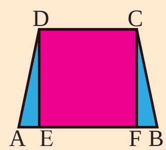
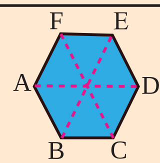
What do you observe from these figures?

In Fig. 2.19 (a), triangle is placed over a semicircle. In Fig. 2.19 (b), trapezium is placed over a square etc.

Two or three plane figures placed adjacently to form a new figure. These are 'combined figures'. The above combined figures are Juxtaposition of some known figures; triangle, rectangle, semi-circle, etc.

> Some combinations of plane figures placed adjacently, with one side equal in length to a side of the other is called a **Juxtaposition** of figures.

Can we see some examples?

S. No.	Plane figures	Juxtaposition	
1.	Two scalene triangles	Quadrilateral	
2.	Two right triangles and a rectangle	Trapezium	
3.	Six equilateral triangles	Hexagon	

(a) Polygon

A **polygon** is a closed plane figure formed by '*n*' line segments.

A plane figure bounded by straight line segments is a **rectilinear figure**.

A rectilinear figure of three sides is called a triangle and four sides is called a **Quadrilateral**.

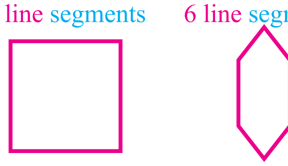


Fig. 2.20

> The word **'Polygon'** means a rectilinear figure with three or more sides.

(b) Regular polygon

If all the sides and angles of a polygon are equal, it is called a **regular polygon**. For example,

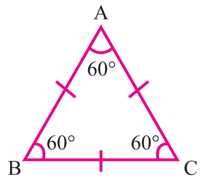


Fig. 2.21

(i) An equilateral triangle is a regular polygon with three sides.

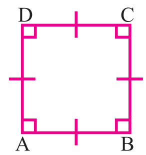


Fig. 2.22

(ii) Square is a regular polygon with four sides.

(c) Irregular polygon

Polygons not having regular geometric shapes are called **irregular polygons**.

(d) Concave polygon

A polygon in which atleast one angle is more than 180°, is called a **concave polygon**.

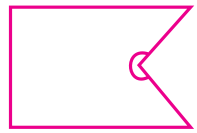


Fig. 2.23

(e) Convex polygon

A polygon in which each interior angle is less than 180°, is called a **convex polygon**.

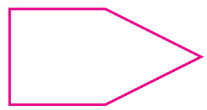


Fig. 2.24

Polygons are classified as follows.

Number of sides	Name of the polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

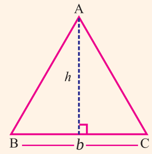
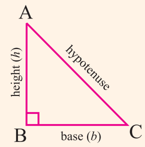
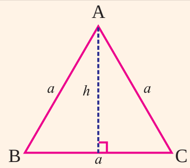
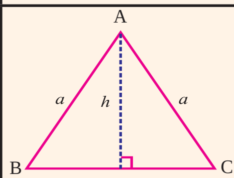
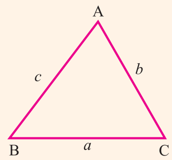
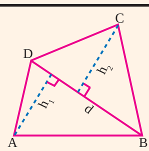
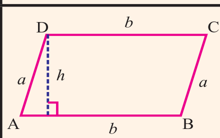
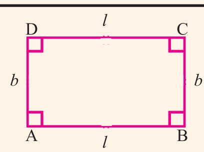
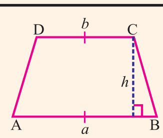
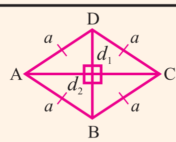
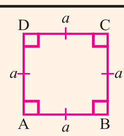
Vijay has fenced his land with 44m barbed wire. Which of the following shape will occupy the maximum area of the land?

a) Circle b) Square

c) Rectangle 2m × 20m

d) Rectangle 7m × 15m

Most of the combined figures are irregular polygons. We divide them into known plane figures. Thus, we can find their areas and perimeters by applying the formulae of plane figures which we have already learnt in class VII. These are listed in the following table.

No.	Name of the Figure	Figure	Area (A) (sq. units)	Perimeter (P) (units)
1.	Triangle		$\frac{1}{2} \times b \times h$	AB + BC + CA
2.	Right triangle		$\frac{1}{2} \times b \times h$	(base + height + hypotenuse)
3.	Equilateral triangle		$\frac{\sqrt{3}}{4}a^2$ where ($\sqrt{3} \simeq 1.732$)	AB+BC+CA = 3a ; Altitude, $h = \frac{\sqrt{3}}{2}a$ units
4.	Isosceles triangle		$h \times \sqrt{a^2 - h^2}$	$2a + 2\sqrt{a^2 - h^2}$
5.	Scalene triangle		$\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$	AB + BC + CA $= (a+b+c)$
6.	Quadrilateral		$\frac{1}{2} \times d \times (h_1 + h_2)$	AB + BC + CD + DA
7.	Parallelogram		$b \times h$	$2 \times (a + b)$
8.	Rectangle		$l \times b$	$2 \times (l + b)$
9.	Trapezium		$\frac{1}{2} \times h \times (a+b)$	AB + BC + CD + DA
10.	Rhombus		$\frac{1}{2} \times d_1 \times d_2$ where d_1, d_2 are diagonals	$4a$
11.	Square		a^2	$4a$

Activity

Divide the given shapes into plane figures as you like and discuss among yourselves.

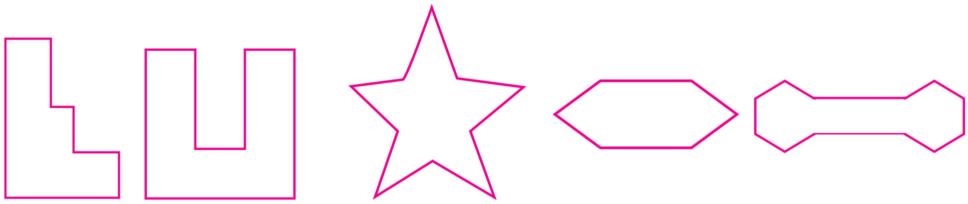


Fig. 2.25

Example 2.5

Find the perimeter and area of the following combined figures.

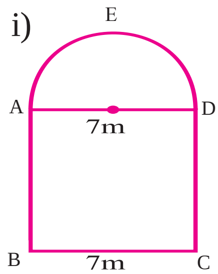
i) 

Fig. 2.26

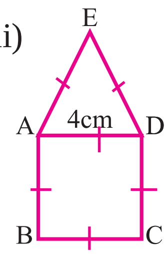
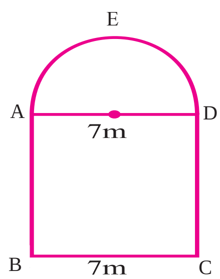
(ii) 

Fig. 2.27

Solution

(i) It is a combined figure made up of a square ABCD and a semicircle DEA. Here, arc $\overset{\frown}{DEA}$ is half the circumference of a circle whose diameter is AD.



Given: Side of a square $= 7$ m

$\therefore$ Diameter of a semicircle $= 7$ m

$\therefore$ Radius of a semicircle, $r = \frac{7}{2}$ m

Perimeter of the combined figure $= \overline{AB} + \overline{BC} + \overline{CD} + \overset{\frown}{DEA}$

$$P = 7 + 7 + 7 + \frac{1}{2} \times \text{(circumference of a circle)}$$

$$= 21 + \frac{1}{2} \times 2\pi r = 21 + \frac{22}{7} \times \frac{7}{2}$$

$$P = 21 + 11 = 32 \text{ m}$$

$\therefore$ Perimeter of the combined figure $= 32$ m.

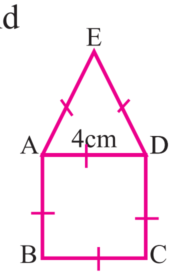
Area of the combined figure $=$ Area of a semicircle $+$ Area of a square

$$A = \frac{\pi r^2}{2} + a^2$$

$$= \frac{22}{7 \times 2} \times \frac{7 \times 7}{2 \times 2} + 7^2 = \frac{77}{4} + 49$$

$\therefore$ Area of the given combined figure $= 19.25 + 49 = 68.25 \text{ m}^2$.

(ii) The given combined figure is made up of a square ABCD and an equilateral triangle DEA.



Given: Side of a square = 4 cm

$\therefore$ Perimeter of the combined figure = AB + BC + CD + DE + EA

= 4 + 4 + 4 + 4 + 4 = 20 cm

$\therefore$ Perimeter of the combined figure = 20 cm.

Area of the given combined figure = Area of a square + Area of an equilateral triangle

$= a^2 + \frac{\sqrt{3}}{4}a^2$ $\quad \sqrt{3} = 1.732$

$= 4 \times 4 + \frac{\sqrt{3}}{4} \times 4 \times 4$

$= 16 + 1.732 \times 4$

Area of the given combined figure = 16 + 6.928 = 22.928

Area of the given figure $\simeq$ 22. 93 cm².

Example 2.6

Find the perimeter and area of the shaded portion

(i)

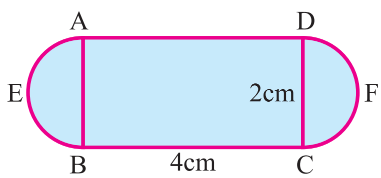


Fig. 2.28

(ii)

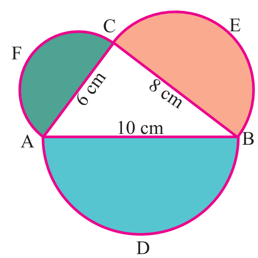
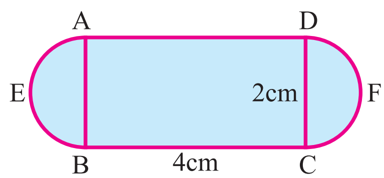


Fig. 2.29

Solution

(i) The given figure is a combination of a rectangle ABCD and two semicircles AEB and DFC of equal area.



Given: Length of the rectangle, l = 4 cm

Breadth of the rectangle, b = 2 cm

Diameter of a semicircle = 2 cm

$\therefore$ Radius of a semicircle, r = $\frac{2}{2}$ = 1 cm

$\therefore$ Perimeter of the given figure = AD+BC+ $\overset{\frown}{AEB}$ + $\overset{\frown}{DFC}$

$= 4 + 4 + 2 \times \frac{1}{2} \times$ (circumference of a circle)

$= 8 + 2 \times \frac{1}{2} \times 2\pi r$

$= 8 + 2 \times \frac{22}{7} \times 1$

$= 8 + 2 \times 3.14$

$= 8 + 6.28$

$\therefore$ Perimeter of the given figure = 14.28 cm.

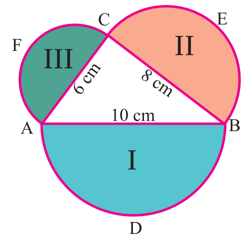
Area of the given figure = Area of a rectangle ABCD + 2 × Area of a semicircle

$$= l \times b + 2 \times \frac{\pi r^2}{2}$$

$$= 4 \times 2 + 2 \times \frac{22 \times 1 \times 1}{7 \times 2}$$

$\therefore$ Total area = 8 + 3. 14 = 11. 14 cm^2.

(ii) Let ADB, BEC and CFA be the three semicircles I, II and III respectively.



Given:

Radius of a semicircle I, $r_1 = \frac{10}{2} = 5$ cm

Radius of a semicircle II, $r_2 = \frac{8}{2} = 4$ cm

Radius of a semicircle III, $r_3 = \frac{6}{2} = 3$ cm

Perimeter of the shaded portion = Perimeter of a semicircle I + Perimeter of a semicircle II + Perimeter of a semicircle III

$$= (\pi + 2) \times 5 + (\pi + 2) \times 4 + (\pi + 2) \times 3$$

$$= (\pi + 2)(5 + 4 + 3) = (\pi + 2) \times 12$$

$$= \left(\frac{22 + 14}{7}\right) \times 12 = \frac{36}{7} \times 12 = 61.714$$

Perimeter of the shaded portion $\simeq$ 61.71cm.

Area of the shaded portion, A = Area of a semicircle I + Area of a semicircle II + Area of a semicircle III

$$A = \frac{\pi r_1^2}{2} + \frac{\pi r_2^2}{2} + \frac{\pi r_3^2}{2}$$

$$= \frac{22}{7 \times 2} \times 5 \times 5 + \frac{22}{7 \times 2} \times 4 \times 4 + \frac{22}{7 \times 2} \times 3 \times 3$$

$$A = \frac{275}{7} + \frac{176}{7} + \frac{99}{7} = \frac{550}{7} = 78.571 \text{ cm}^2$$

Area of the shaded portion $\simeq$ 78.57 cm^2

In this example we observe that,

Area of semicircle BEC + Area of semicircle CFA = Area of semicircle ADB

MATHEMATICS

Example 2.7

A horse is tethered to one corner of a rectangular field of dimensions 70 m by 52 m by a rope 28 m long for grazing. How much area can the horse graze inside? How much area is left ungrazed?

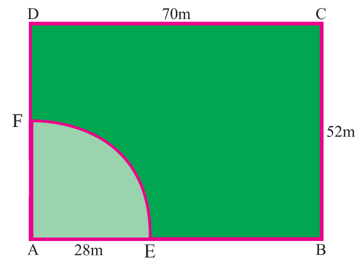


Fig. 2.30

Solution

Length of the rectangle, l = 70 m

Breadth of the rectangle, b = 52 m

Length of the rope = 28 m

Shaded portion AEF indicates the area in which the horse can graze. Clearly, it is the area of a quadrant of a circle of radius, r = 28 m

$$\text{Area of the quadrant AEF} = \frac{1}{4}\times \pi r^2 \text{ sq. units}$$

$$= \frac{1}{4}\times\frac{22}{7}\times 28\times 28 = 616 \text{ m}^2$$

$$\therefore \text{ Grazing Area} = 616 \text{ m}^2.$$

Area left ungrazed = Area of the rectangle ABCD – Area of the quadrant AEF

$$\text{Area of the rectangle ABCD} = l \times b \text{ sq. units}$$

$$= 70 \times 52 = 3640 \text{ m}^2$$

$$\therefore \text{ Area left ungrazed} = 3640 - 616 = 3024 \text{ m}^2.$$

Example 2.8

In the given figure, ABCD is a square of side 14 cm. Find the area of the shaded portion.

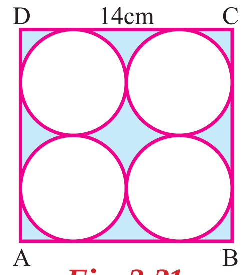


Fig. 2.31

Solution

Side of a square, a = 14 cm

Radius of each circle, $r = \frac{7}{2}$ cm

Area of the shaded portion = Area of a square – 4 × Area of a circle

$$= a^2 - 4\,(\pi r^2)$$

$$= 14 \times 14 - 4 \times \frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$

$$= 196 - 154$$

$$\therefore \text{ Area of the shaded portion} = 42 \text{ cm}^2.$$

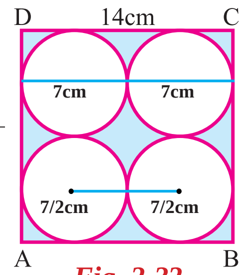


Fig. 2.32

Example 2.9

A copper wire is in the form of a circle with radius 35 cm. It is bent into a square. Determine the side of the square.

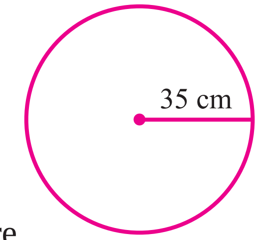


Fig. 2.33

Solution

Given: Radius of a circle, r = 35 cm.

Since the same wire is bent into the form of a square,

$$\text{Perimeter of the circle} = \text{Perimeter of the square}$$

$$\text{Perimeter of the circle} = 2\pi r \text{ units}$$

$$= 2 \times \frac{22}{7} \times 35 \text{ cm}$$

$$P = 220 \text{ cm.}$$

Let 'a' be the side of a square.

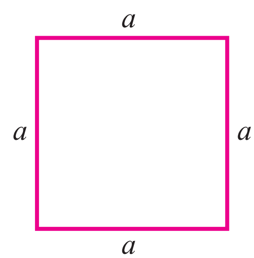


Fig. 2.34

$$\text{Perimeter of a square} = 4a \text{ units}$$

$$4a = 220$$

$$a = 55 \text{ cm}$$

$$\therefore \text{Side of the square} = 55 \text{ cm.}$$

Example 2.10

Four equal circles are described about four corners of a square so that each touches two of the others as shown in the Fig. 2.35. Find the area of the shaded portion, each side of the square measuring 28 cm.

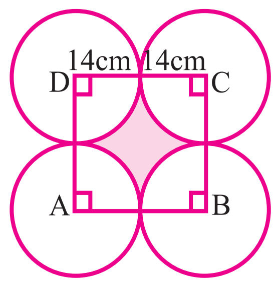


Fig. 2.35

Solution

Let ABCD be the given square of side a.

$$\therefore a = 28\text{cm}$$

$$\therefore \text{Radius of each circle, } r = \frac{28}{2}$$

$$= 14 \text{ cm}$$

$$\text{Area of the shaded portion} = \text{Area of a square} - 4 \times \text{Area of a quadrant}$$

$$= a^2 - 4 \times \frac{1}{4} \times \pi r^2$$

$$= 28 \times 28 - 4 \times \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$$

$$= 784 - 616$$

$$\therefore \text{Area of the shaded portion} = 168 \text{ cm}^2.$$

Example 2.11

A 14 m wide athletic track consists of two straight sections each 120 m long joined by semi-circular ends with inner radius is 35 m. Calculate the area of the track.

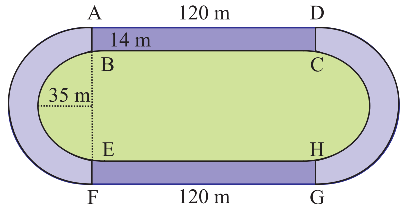


Fig. 2.36

Solution

Given: Radius of the inner semi circle, r = 35 m

Width of the track = 14 m

$\therefore$ Radius of the outer semi circle, R = 35 + 14 = 49 m

R = 49 m

Area of the track is the sum of the areas of the semicircular tracks and the areas of the rectangular tracks.

Area of the rectangular tracks ABCD and EFGH = $2 \times (l \times b)$

$= 2 \times 14 \times 120 = 3360 \text{ m}^2$.

Area of the semicircular tracks = 2 × (Area of the outer semicircle – Area of the inner semicircle)

$$= 2 \times \left(\frac{1}{2}\pi R^2 - \frac{1}{2}\pi r^2\right)$$

$$= 2 \times \frac{1}{2} \times \pi (R^2 - r^2)$$

$$= \frac{22}{7} \times (49^2 - 35^2) \quad (\because a^2 - b^2 = (a+b)(a-b))$$

$$= \frac{22}{7}(49 + 35)(49 - 35)$$

$$= \frac{22}{7} \times 84 \times 14 = 3696 \text{ m}^2$$

$\therefore$ Area of the track $= 3360 + 3696 = 7056 \text{ m}^2$.

Example 2.12

In the given Fig. 4.37, PQSR represents a flower bed. If OP = 21 m and OR = 14 m, find the area of the shaded portion.

Solution

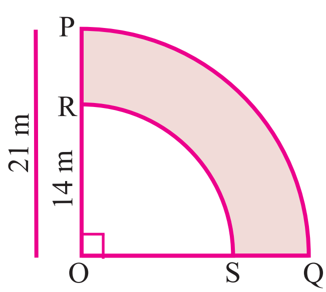


Fig. 2.37

Given : OP = 21 m and OR = 14 m

Area of the flower bed = Area of the quadrant OQP – Area of the quadrant OSR

$$= \frac{1}{4}\pi \times OP^2 - \frac{1}{4}\pi \times OR^2$$

$$= \frac{1}{4}\times\pi\times 21^2 - \frac{1}{4}\times\pi\times 14^2$$

$$= \frac{1}{4}\times\pi\times(21^2 - 14^2)$$

$$= \frac{1}{4}\times\frac{22}{7}\times(21+14)\times(21-14)$$

$\therefore$ Area of the flower bed $= \frac{1}{4}\times\frac{22}{7}\times 35\times 7 = 192.5\ \text{m}^2$.

Example 2.13

Find the area of the shaded portions in the Fig. 2.38, where ABCD is a square of side 7 cm.

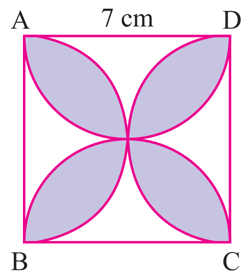


Fig. 2.38

Solution

Let us mark the unshaded portions by I, II, III and IV as shown in the Fig. 2.39.

Let P,Q,R and S be the mid points of AB, BC,CD and DA respectively.

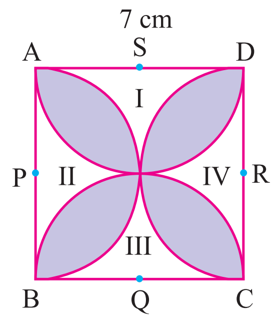


Fig. 2.39

Side of the square, a = 7 cm

Radius of the semicircle, $r = \frac{7}{2}$ cm

Area of I + Area of III = Area of a square ABCD – Area of two semicircles with centres P and R

$$= a^2 - 2\times\frac{1}{2}\times\pi r^2$$

$$= 7\times 7 - 2\times\frac{1}{2}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}$$

$\therefore$ Area of I + Area of III $= \left(49 - \frac{77}{2}\right)\text{cm}^2 = \frac{21}{2}\text{cm}^2$.

Similarly, we have

Area of II + Area of IV $= \left(49 - \frac{77}{2}\right)\text{cm}^2 = \frac{21}{2}\text{cm}^2$.

Area of the shaded portions = Area of the square ABCD – (Area of I + Area of II + Area of III + Area of IV)

$$= 49 - \left(\frac{21}{2} + \frac{21}{2}\right)$$

$$= 49 - 21 = 28\ \text{cm}^2$$

$\therefore$ Area of the shaded portions = 28 cm^2.

Example 4.14

A surveyor has sketched the measurements of a land as below. Find the area of the land.

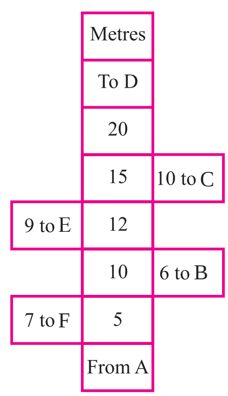
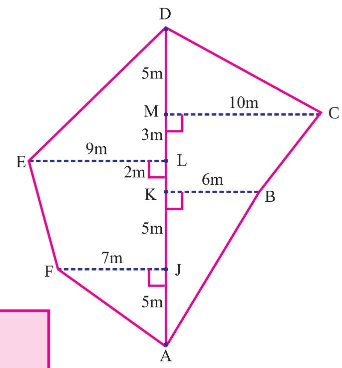


Fig. 2.40

Solution

Let J, K, L, M be the surveyor's marks from A to D.

Given: AJ = 5 m , JF = 7 m,
KB = 6 m, LE = 9 m , MC = 10 m,
AK = 10 m, AL = 12 m,
AM = 15 m and AD = 20 m.



The given land is the combination of the trapezium KBCM, LEFJ and right angled triangles ABK, MCD, DEL and JFA.

Area of the trapezium = $\frac{1}{2} \times h(a+b)$ sq. units

Let A_1 denote the area of the trapezium KBCM.

$$A_1 = \frac{1}{2} \times (KB + MC) \times KM$$
$$= \frac{1}{2} \times (6+10) \times 5$$
$$A_1 = \frac{1}{2} \times 16 \times 5 = 40 \text{ m}^2.$$

($\because$ parallel sides are KB, MC and height is KM
KB = 6 m, MC = 10 m,
KM = AM – AK
= 15 – 10 = 5 m)

Let A_2 denote the area of the trapezium LEFJ.

$$A_2 = \frac{1}{2} \times (JF + LE) \times JL$$
$$= \frac{1}{2} \times (7+9) \times 7$$
$$A_2 = \frac{1}{2} \times 16 \times 7 = 56 \text{ m}^2.$$

($\because$ parallel sides are LE, JF and height is JL
JF = 7 m, LE = 9 m,
JL = AL – AJ
= 12 – 5 = 7 m)

Let A_3 denote the area of the right angled triangle ABK.

$$A_3 = \frac{1}{2} \times AK \times KB$$
$$A_3 = \frac{1}{2} \times 10 \times 6 = 30 \text{ m}^2.$$

Let A_4 denote the area of the right angled triangle MCD.

$$A_4 = \frac{1}{2} \times MC \times MD.$$
$$= \frac{1}{2} \times 10 \times 5$$
$$A_4 = \frac{50}{2} = 25 \text{ m}^2.$$

Let A_5 denote the area of the right angled triangle DEL.

$$A_5 = \frac{1}{2} \times DL \times LE$$
$$= \frac{1}{2} \times (AD - AL) \times LE$$
$$= \frac{1}{2}(20 - 12) \times 9$$
$$A_5 = \frac{1}{2} \times 8 \times 9 = 36 \text{ m}^2.$$

Let A_6 denote the area of the right angled triangle JFA.

$$A_6 = \frac{1}{2} \times AJ \times JF$$

$$= \frac{1}{2} \times 5 \times 7 = \frac{35}{2} = 17.5\ m^2.$$

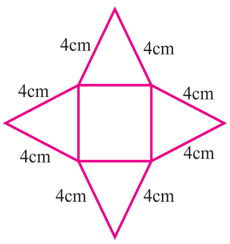
$$\text{Area of the land} = A_1 + A_2 + A_3 + A_4 + A_5 + A_6$$

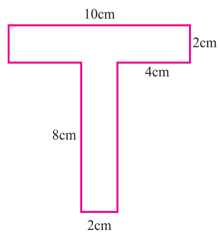
$$= 40 + 56 + 30 + 25 + 36 + 17.5$$

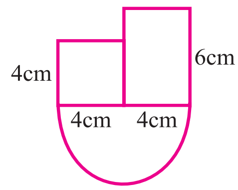
$$\therefore \text{Area of the land} = 204.5\ m^2.$$

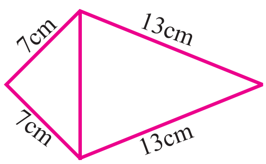
EXERCISE 2.2

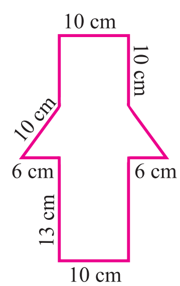
1. Find the perimeter of the following figures

(i)


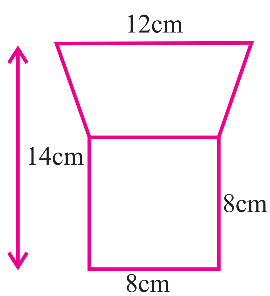
(ii)


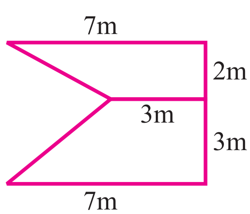
(iii)


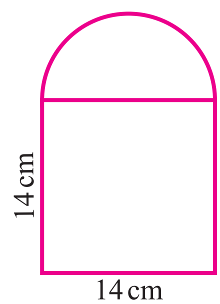
(iv)


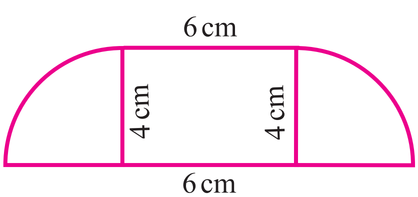
(v)


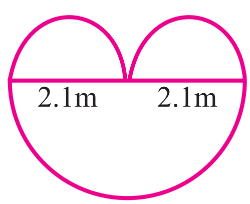
2. Find the area of the following figures

(i)


(ii)


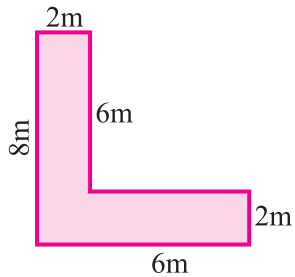
(iii)


(iv)


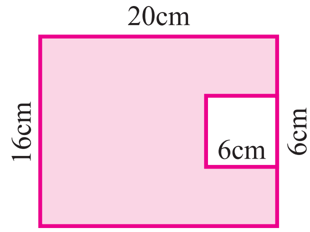
(v)


3. Find the area of the coloured regions

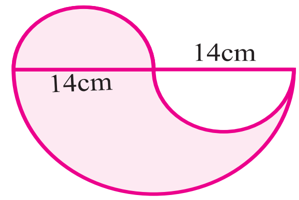
(i)



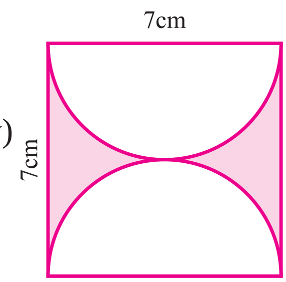
(ii)



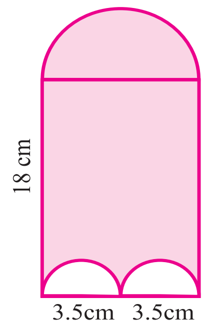
(iii)



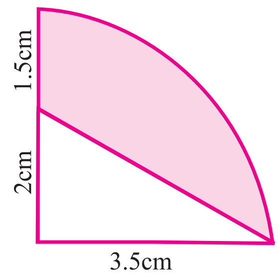
(iv)



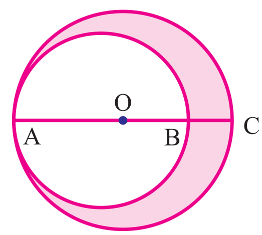
(v)



(vi)

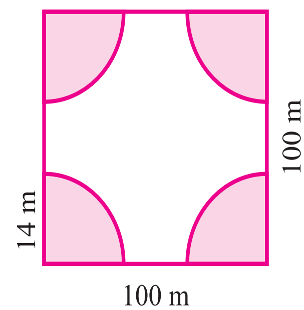


4. In the given figure, find the area of the shaded portion if AC = 54 cm, BC = 10 cm, and O is the centre of bigger circle.

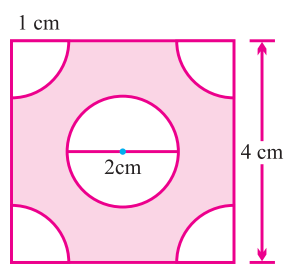


5. A cow is tied up for grazing inside a rectangular field of dimensions 40 m $\times$ 36 m in one corner of the field by a rope of length 14 m. Find the area of the field left ungrazed by the cow.

6. A square park has each side of 100 m. At each corner of the park there is a flower bed in the form of a quadrant of radius 14 m as shown in the figure. Find the area of the remaining portion of the park.

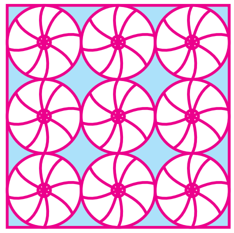


7. Find the area of the shaded region shown in the figure. The four corners are quadrants. At the centre, there is a circle of diameter 2 cm.



8. A paper is in the form of a rectangle ABCD in which AB = 20 cm and BC = 14 cm. A semicircular portion with BC as diameter is cut off. Find the area of the remaining part.

9. On a square handkerchief, nine circular designs each of radius 7 cm are made. Find the area of the remaining portion of the handkerchief.



10. From each of the following notes in the field book of a surveyor, make a rough plan of the field and find its area.

(i)

	Metres	
	To D	
	240	
	210	30 to C
40 to E	170	
60 to E	130	
	70	50 to B
	From A	

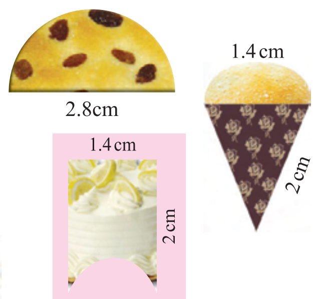
(ii)

	To J	
	1000 K	
300 M	600 I	
	300 H	800 L
	From G	

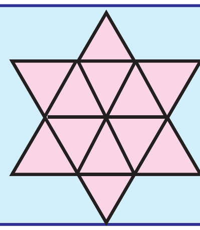
Activity

Can you help the ant?

An ant is moving around a few food pieces of different shapes scattered on the floor. For which food-piece would the ant have to take a shorter round and longer round?

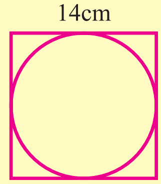


How many triangles are there?

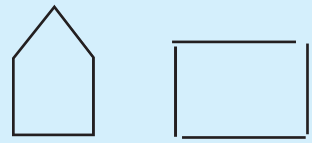


Try these

Which is smaller? The perimeter of a square or the perimeter of a circle inscribed in it?



Which one of these figures has perimeter ?



(a) (b)

Concept Summary

- The central angle of a circle is 360°.
- Perimeter of a semicircle $= (\pi + 2) \times r$ units.
- Area of a semicircle $= \dfrac{\pi r^2}{2}$ sq . units.
- The central angle of a semicircle is 180°.
- Perimeter of a quadrant $= \left(\dfrac{\pi}{2} + 2\right) \times r$ units.
- Area of a quadrant $= \dfrac{\pi r^2}{4}$ sq . units.
- The central angle of a quadrant is 90°.
- Perimeter of a combined figure is length of its boundary.
- A polygon is a closed plane figure formed by 'n' line segments.
- Regular polygons are polygons in which all the sides and angles are equal.
- Irregular polygons are combination of plane figures.

Geometry

3

3.1 Introduction

Geometry was developed by Egyptians more than 1000 years before Christ, to help them mark out their fields after the floods from the Nile. But it was abstracted by the Greeks into logical system of proofs with necessary basic postulates or axioms.

Geometry plays a vital role in our life in many ways. In nature, we come across many geometrical shapes like hexagonal bee-hives, spherical balls, rectangular water tanks, cylindrical wells and so on. The construction of Pyramids is a glaring example for practical application of geometry. Geometry has numerous practical applications in many fields such as Physics, Chemistry, Designing, Engineering, Architecture and Forensic Science.

The word 'Geometry' is derived from two Greek words 'Geo' which means 'earth' and 'metro' which means 'to measure'. Geometry is a branch of mathematics which deals with the shapes, sizes, positions and other properties of the object.

In class VII, we have learnt about the properties of parallel lines, transversal lines, angles in intersecting lines, adjacent and alternative angles. Moreover, we have also come across the angle sum property of a triangle.

Euclid
Father of Geometry

"Euclid was a great Greek Mathematician who gave birth to logical thinking in geometry". Euclid collected the various information on geometry around 300B.C. and published them in the form of 13 books in a systematic manner. These books are called Euclid Elements.

Euclid said : "The whole is greater with any of its parts".

Let us recall the results through the following exercise.

REVISION EXERCISE

1. In Fig.3.1, $x° = 128°$. Find $y°$.

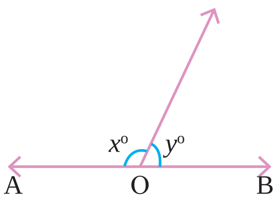


Fig. 3.1

2. Find ∠BCE and ∠ECD in the Fig.3.2, where ∠ACD = 90°

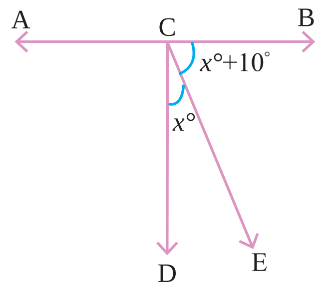


Fig. 3.2

3. Two angles of a triangle are 43° and 27°. Find the third angle.

4. Find $x°$ in the Fig.3.3, if PQ || RS.

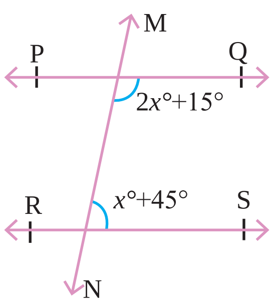


Fig. 3.3

5. In the Fig.3.4, two lines AB and CD intersect at the point O. Find the value of $x°$ and $y°$.

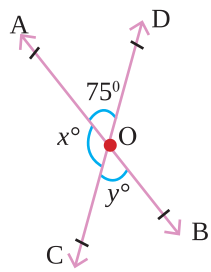


Fig. 3.4

6. In the Fig. 3.5 AB || CD. Fill in the blanks.
   (i) ∠EFB and ∠FGD are .... angles.
   (ii) ∠AFG and ∠FGD are .... angles.
   (iii) ∠AFE and ∠FGC are .... angles.

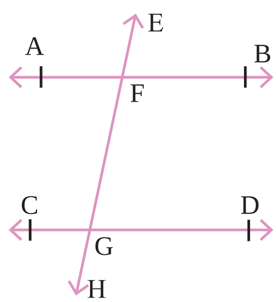


Fig. 3.5

3.2 Properties of Triangles

A triangle is a closed figure bounded by three line segments in a plane.

Triangle can be represented by the notation 'Δ'.

In any triangle ABC, the sides opposite to the vertices A, B, C can be represented by a, b, c respectively.

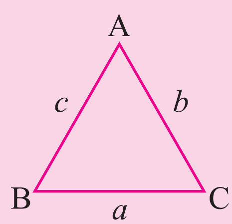


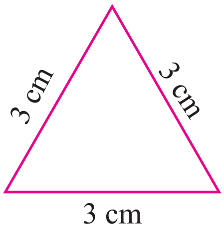
Fig. 3.6

3.2.1. Kinds of Triangles

Triangles can be classified into two types based on sides and angles.

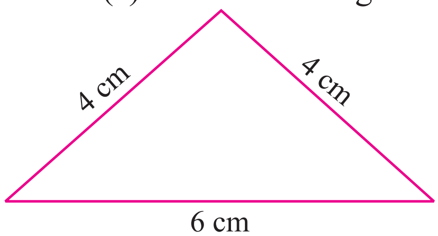
Based on sides:

(a) Equilateral Triangle



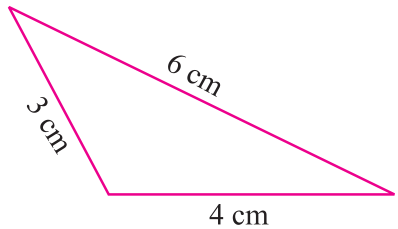
Three sides are equal

(b) Isosceles Triangle



Two sides are equal

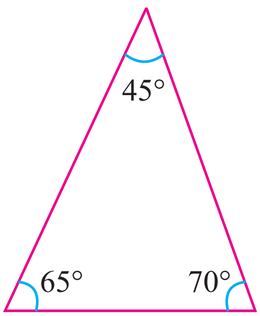
(c) Scalene Triangle



All sides are different

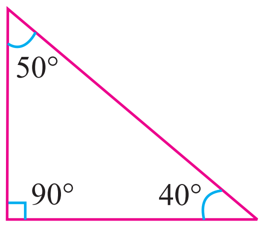
Based on angles:

(d) Acute Angled Triangle



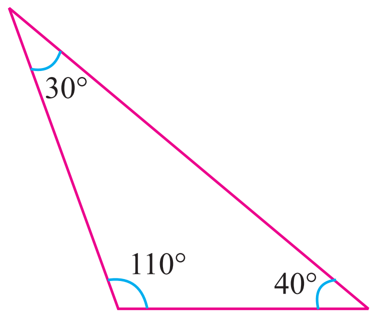
Three acute angles

(e) Right Angled Triangle



One right angle

(f) Obtuse Angled Triangle



One obtuse angle

3.2.2 Angle Sum Property of a Triangle

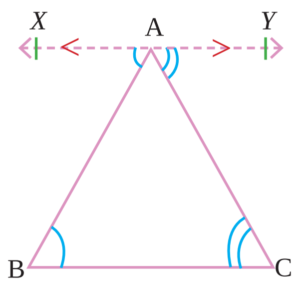


Fig. 3.7

Theorem 1

The sum of the three angles of a triangle is 180°.

Given : ABC is a Triangle.

To Prove : $\angle ABC + \angle BCA + \angle CAB = 180^{\circ}$

Construction: Through the vertex A draw XY parallel to BC.

Proof :

Statement	Reason
(i) $BC \parallel XY$ and AB is a transversal $\therefore \angle ABC = \angle XAB$	Alternate angles.
(ii) AC is a transversal, $\angle BCA = \angle YAC$	Alternate angles.
(iii) $\angle ABC + \angle BCA = \angle XAB + \angle YAC$	By adding (i) and (ii).
(iv) $(\angle ABC + \angle BCA) + \angle CAB =$ $(\angle XAB + \angle YAC) + \angle CAB$	By adding $\angle BAC$ on both sides.
(v) $\therefore \angle ABC + \angle BCA + \angle CAB = 180^{\circ}$	The angle of a straight line is 180°.

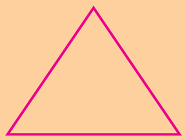
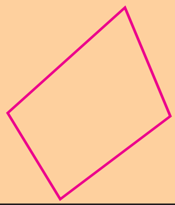
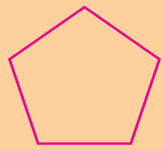
Results

(i) Triangle is a polygon of three sides.

(ii) Any polygon could be divided into triangles by joining the diagonals.

(iii) The sum of the interior angles of a polygon can be given by the formula $(n-2)\,180°$, where n is the number of sides.

Activity

Figure			
Number of sides	3	4	5
Classification	Triangle	Quadrilateral	Pentagon
Sum of angles			

Theorem 2

If a side of a triangle is produced, the exterior angle so formed, is equal to the sum of the two interior opposite angles.

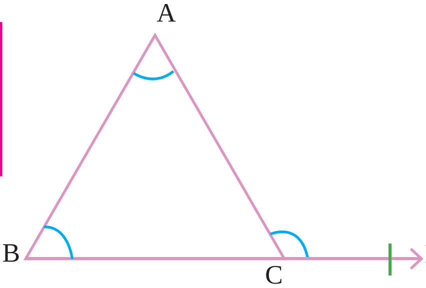


Fig. 3.8

Given : ABC is a triangle.
BC is produced to D.

To Prove : $\angle ACD = \angle ABC + \angle CAB$

Proof :

Statement	Reason
(i) In ΔABC, $\angle ABC + \angle BCA + \angle CAB = 180^0$	Angle sum property of a triangle.
(ii) $\angle BCA + \angle ACD = 180^0$	Sum of the adjacent angles of a straight line.
(iii) $\angle ABC + \angle BCA + \angle CAB = \angle BCA + \angle ACD$	Equating (i) and (ii).
(iv) $\therefore \angle ABC + \angle CAB = \angle ACD$	Subtracting $\angle BCA$ on both sides of (iii).
(v) The exterior angle $\angle ACD$ is equal to the sum of the interior opposite angles $\angle ABC$ and $\angle CAB$.	Hence proved.

Results

(i) In a traingle the angles opposite to equal sides are equal.

(ii) In a traingle the angle opposite to the longest side is largest.

Example 3.1

In $\Delta ABC, \angle A = 75^\circ, \angle B = 65^\circ$ find $\angle C$.

Solution

We know that in ΔABC,

$$\angle A + \angle B + \angle C = 180^\circ$$
$$75^\circ + 65^\circ + \angle C = 180^\circ$$
$$140^\circ + \angle C = 180^\circ$$
$$\angle C = 180^\circ - 140^\circ$$
$$\therefore \angle C = 40^\circ.$$

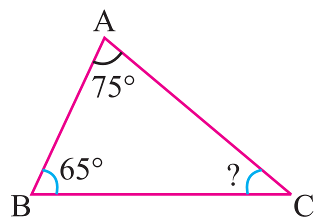


Fig. 3.9

Example 3.2

In ΔABC, given that $\angle A = 70^\circ$ and AB = AC. Find the other angles of Δ ABC.

Solution

Let $\angle B = x^\circ$ and $\angle C = y^\circ$.

Given that ΔABC is an isosceles triangle.

$$AC = AB$$
$$\angle B = \angle C \quad \text{[Angles opposite to equal sides are equal]}$$
$$x^\circ = y^\circ$$

In $\Delta ABC, \angle A + \angle B + \angle C = 180^\circ$

$$70^\circ + x^\circ + y^\circ = 180^\circ$$
$$70^\circ + x^\circ + x^\circ = 180^\circ \quad [\because x^\circ = y^\circ]$$
$$2x^\circ = 180^\circ - 70^\circ$$
$$2x^\circ = 110^\circ$$
$$x^\circ = \frac{110^\circ}{2} = 55^\circ.$$ Hence $\angle B = 55^\circ$ and $\angle C = 55^\circ$.

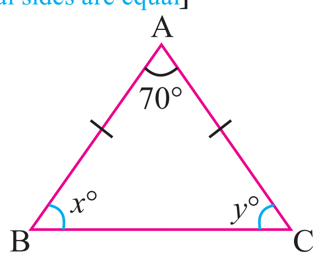


Fig. 3.10

Example 3.3

The measures of the angles of a triangle are in the ratio 5 : 4 : 3. Find the angles of the triangle.

Solution

Given that in a ΔABC, $\angle A : \angle B : \angle C = 5 : 4 : 3$.

Let the angles of the given triangle be $5x^\circ$, $4x^\circ$ and $3x^\circ$.

We know that the sum of the angles of a triangle is 180° .

$$5x^\circ + 4x^\circ + 3x^\circ = 180^\circ \Rightarrow 12x^\circ = 180^\circ$$

$$x^\circ = \frac{180^0}{12} = 15^\circ$$

So, the angles of the triangle are 75°, 60° and 45°.

Example 3.4

Find the angles of the triangle ABC, given in Fig.3.11.

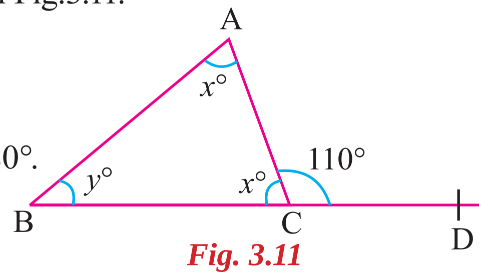


Fig. 3.11

Solution

BD is a straight line.

We know that angle in the line segment is 180°.

$$x^\circ + 110^\circ = 180^\circ$$

$$x^\circ = 180^\circ - 110^\circ$$

$$x^\circ = 70^\circ$$

We know that the exterior angle is equal to the sum of the two interior opposite angles.

$$x^\circ + y^\circ = 110^\circ$$

$$70^\circ + y^\circ = 110^\circ$$

$$y^\circ = 110^\circ - 70^\circ = 40^\circ$$

Hence, $x^\circ = 70^\circ$

and $y^\circ = 40^\circ$.

Example 3.5

Find the value of $\angle DEC$ from the given Fig. 3.12.

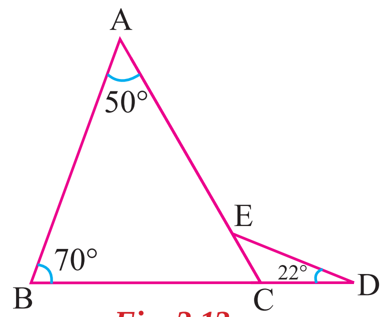


Fig. 3.12

Solution

We know that in any triangle, exterior angle is equal to the sum of the interior angles opposite to it.

In ΔABC, $\angle ACD = \angle ABC + \angle CAB$

$\therefore \angle ACD = 70^\circ + 50^\circ = 120^\circ$

Also, $\angle ACD = \angle ECD = 120^\circ$.

Considering ΔECD,

$$\angle ECD + \angle CDE + \angle DEC = 180^0 \quad \text{[Sum of the angles of a triangle]}$$

$$120^0 + 22^0 + \angle DEC = 180^0$$

$$\angle DEC = 180^\circ - 142^\circ$$

$$\angle DEC = 38^\circ$$

Activity

Draw all the types of triangles T_1, T_2, T_3, T_4, T_5 and T_6. Let us name the triangles as ABC.Let a, b, c be the sides opposite to the vertices A, B, C respectively. Measure the sides and arrange the data as follows:

Serial No.of Δ	a (cm)	b (cm)	c (cm)	$(c+a) > b$ True / False	$(a + b) > c$ True / False	$(b + c) > a$ True / False
T_1						
T_2						
T_3						
T_4						
T_5						
T_6						

What do you observe from this table ?

Theorem 3

Any two sides of a triangle together is greater than the third side.

(This is known as Triangle Inequality)

Verification :

Consider the triangle ABC such that BC = 12 cm, AB = 8 cm, AC = 9 cm.

(i)AB = 8 cm , AB + BC = 20 cm

(ii)BC = 12 cm , BC + CA = 21 cm

(iii)CA = 9 cm, CA + AB = 17 cm

Now clearly ,

(i) AB + BC > CA

(ii) BC + CA > AB

(iii) CA + AB > BC

In all the cases, we find that the sum of any two sides of a triangle is greater than the third side.

Activity

Form a triangle using straws of length 3 cm, 4 cm and 5 cm. Similarly try to form triangles of the following length.

a) 5 cm, 7 cm, 11 cm.

b) 5 cm, 7 cm, 14 cm.

c) 5 cm, 7 cm, 12 cm.

Conclude your findings.

Example 3.6

Which of the following will form the sides of a triangle?

(i) 23cm, 17cm, 8cm (ii) 12cm, 10cm, 25cm (iii) 9cm, 7cm, 16cm

Solution

(i) 23cm, 17cm, 8cm are the given lengths.

Here 23 + 17 > 8, 17 + 8 > 23 and 23 + 8 > 17.

$\therefore$ 23cm, 17cm, 8cm will form the sides of a triangle.

(ii) 12cm, 10cm, 25cm are the given lengths.

Here 12 + 10 is not greater than 25. ie, $[12 + 10 \ngtr 25]$

$\therefore$ 12 cm, 10 cm, 25 cm will not form the sides of a triangle.

(iii) 9 cm, 7 cm, 16 cm are given lengths. 9 + 7 is not greater than 16.

ie, $[9 + 7 = 16, 9 + 7 \ngtr 16]$

$\therefore$ 9 cm, 7 cm and 16 cm will not be the sides of a triangle.

Results

(i)	$c + a > b$	$\Longrightarrow$	$b < c + a$	$\Longrightarrow$	$b - c < a$
(ii)	$b + c > a$	$\Longrightarrow$	$a < b + c$	$\Longrightarrow$	$a - b < c$
(iii)	$a + b > c$	$\Longrightarrow$	$c < a + b$	$\Longrightarrow$	$c - a < b$

From the above results we observe that **in any triangle the difference between the length of any two sides is less than the third side.**

EXERCISE 3.1

1. Choose the correct answer:

(i) Which of the following will be the angles of a triangle?

(A) 35°, 45°, 90° (B) 26°, 58°, 96°

(C) 38°, 56°, 96° (D) 30°, 55°, 90°

(ii) Which of the following statement is correct ?

(A) Equilateral triangle is equiangular.

(B) Isosceles triangle is equiangular.

(C) Equiangular triangle is not equilateral.

(D) Scalene triangle is equiangular

(iii) The three exterior angles of a triangle are 130°, 140°, x° then x° is

(A) 90° (B) 100° (C) 110° (D) 120°

(iv) Which of the following set of measurements will form a triangle?

(A) 11 cm, 4 cm, 6 cm (B) 13 cm, 14 cm, 25 cm

(C) 8 cm, 4 cm, 3 cm (D) 5 cm, 16 cm, 5 cm

(v) Which of the following will form a right angled triangle, given that the two angles are

(A) 24°, 66° (B) 36°, 64°

(C) 62°, 48° (D) 68°, 32°

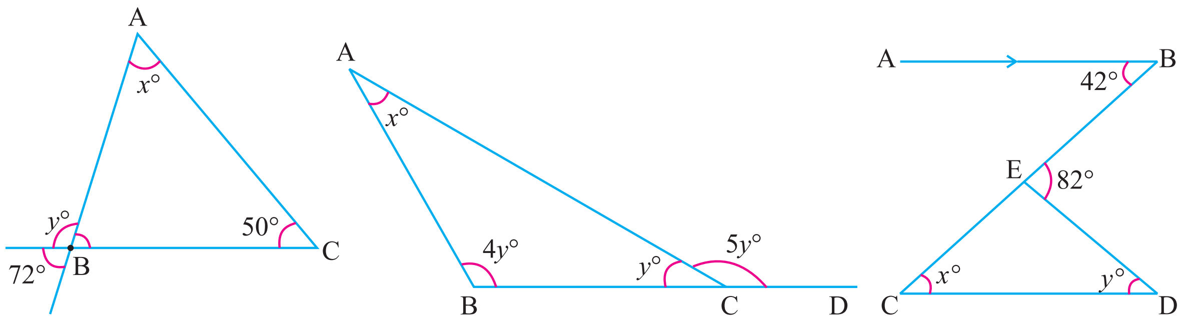
2. The angles of a triangle are $(x - 35)°$, $(x - 20)°$ and $(x + 40)°$. Find the three angles.

3. In ΔABC, the measure of $\angle A$ is greater than the measure of $\angle B$ by 24°. If exterior angle $\angle C$ is 108°. Find the angles of the ΔABC.

4. The bisectors of $\angle B$ and $\angle C$ of a ΔABC meet at O.

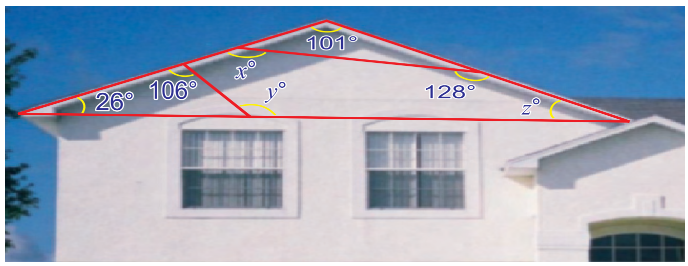
   Show that $\angle BOC = 90^\circ + \frac{\angle A}{2}$.

5. Find the value of x° and y° from the following figures:



(i) (ii) (iii)

6. Find the angles x°, y° and z° from the given figure.



3.3 Congruence of Triangles

We are going to learn the important geometrical idea "Congruence".

To understand what congruence is, we will do the following activity:

Activity

Take two ten rupee notes. Place them one over the other. What do you observe?

One note covers the other completely and exactly.

From the above activity we observe that the figures are of the same shape and the same size.

In general, if two geometrical figures are identical in shape and size then they are said to be congruent.

Activity

Check whether the following objects are congruent or not :

(a) Postal stamps of same denomination.

(b) Biscuits in the same pack.

(c) Shaving blades of same brand.

Now we will consider the following plane figures.

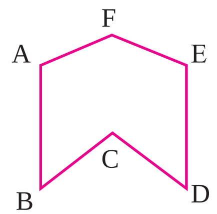


Fig. 3.13

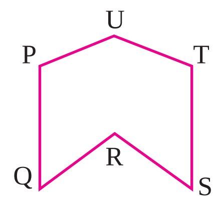


Fig. 3.14

Observe the above two figures. Are they congruent? How to check?

We use **the Method of Superposition.**

Step 1 : Take a trace copy of the Fig. 3.13. We can use Carbon sheet.

Step 2 : Place the trace copy on Fig. 3.14 without bending, twisting and stretching.

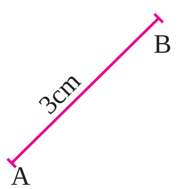
Step 3 : Clearly the figure covers each other completely.

Therefore the two figures are congruent.

Congruent: Two plane figures are Congruent if each when superposed on the other covers it exactly. It is denoted by the symbol "$\equiv$".

3.3.1 (a) Congruence among Line Segments

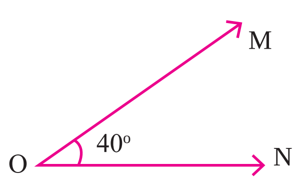
Two line segments are congruent, if they have the same length.

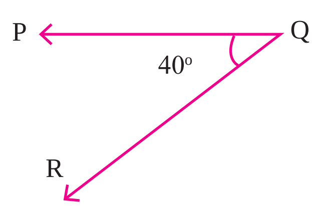


Here, the length of AB = the length of CD. Hence $\overline{AB} \equiv \overline{CD}$

(b) Congruence of Angles

Two angles are congruent, if they have the same measure.

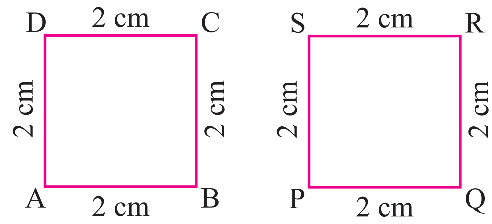




Here the measures are equal. Hence $\angle MON \equiv \angle PQR$.

(c) Congruence of Squares

Two squares having same sides are congruent to each other.

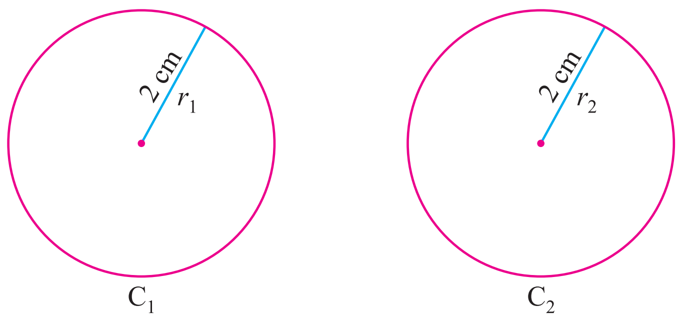


Here, sides of the square ABCD = sides of the square PQRS.

$\therefore$ Square ABCD $\equiv$ Square PQRS

(d) Congruence of Circles

Two circles having the same radius are congruent.

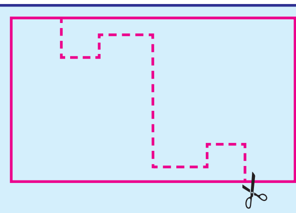


In the given figure, radius of circle C_1 = radius of circle C_2.

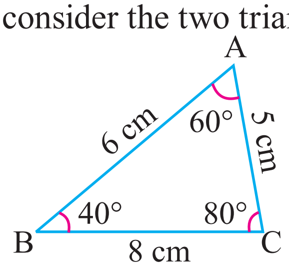
$\therefore$ Circle $C_1 \equiv$ Circle C_2

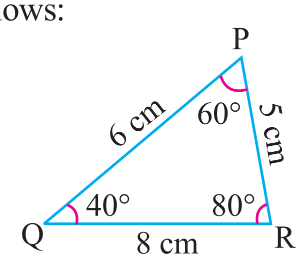
Cut this figure into two pieces through the dotted lines .

What do you understand from these two pieces?



The above congruences motivated us to learn about the congruence of triangles. Let us consider the two triangles as follows:





If we superpose ΔABC on ΔPQR with A on P, B on Q and C on R such that the two triangles cover each other exactly with the corresponding vertices, sides and angles.

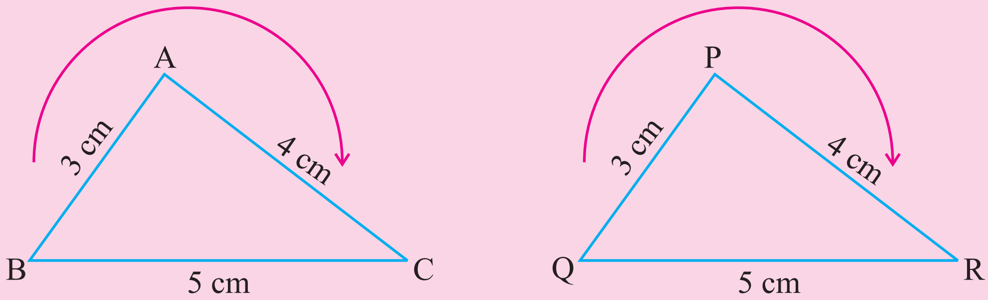
We can match the corresponding parts as follows:

Corresponding Vertices	Corresponding Sides	Corresponding Angles
$A \longleftrightarrow P$	AB = PQ	$\angle A = \angle P$
$B \longleftrightarrow Q$	BC = QR	$\angle B = \angle Q$
$C \longleftrightarrow R$	CA = RP	$\angle C = \angle R$

3.3.2. Congruence of Triangles

Two triangles are said to be congruent, if the three sides and the three angles of one triangle are respectively equal to the three sides and three angles of the other.

Note: While writing the congruence condition between two triangles the order of the vertices is significant.



If $\Delta ABC \equiv \Delta PQR$, then the congruence could be written as follows in different orders $\Delta BAC \equiv \Delta QPR$, $\Delta CBA \equiv \Delta RQP$ and so on. We can also write in anticlockwise direction.

3.3.3. Conditions for Triangles to be Congruent

We know that, if two triangles are congruent, then six pairs of their corresponding parts (Three pairs of sides, three pairs of angles) are equal.

But to ensure that two triangles are congruent in some cases, it is sufficient to verify that only three pairs of their corresponding parts are equal, which are given as axioms.

Axiom: The simple properties which are true without actually proving them.

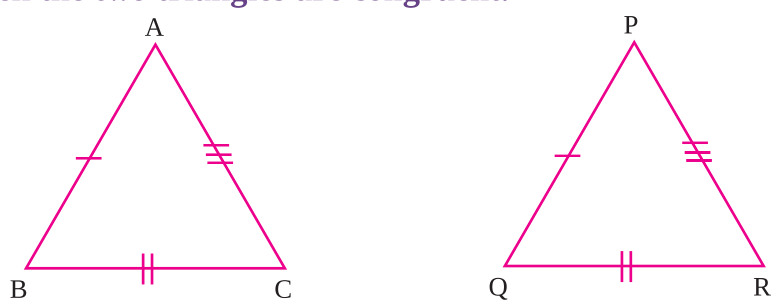
There are four such basic axioms with different combinations of the three pairs of corresponding parts. These axioms help us to identify the congruent triangles.

If 'S' denotes the sides, 'A' denotes the angles, 'R' denotes the right angle and 'H' denotes the hypotenuse of a triangle then the axioms are as follows:

(i) SSS axiom (ii) SAS axiom (iii) ASA axiom (iv) RHS axiom

(i) SSS Axiom (Side-Side-Side axiom)

If three sides of a triangle are respectively equal to the three sides of another triangle then the two triangles are congruent.



We consider the triangles ABC and PQR such that,

AB = PQ, BC = QR and CA= RP.

Take a trace copy of ΔABC and superpose on ΔPQR such that

AB on PQ , BC on QR and AC on PR

Since AB = PQ $\Rightarrow$ A lies on P, B lies on Q

Similarly BC = QR $\Rightarrow$ C lies on R

Now, the two triangles cover each other exactly.

$$\therefore \Delta ABC \equiv \Delta PQR$$

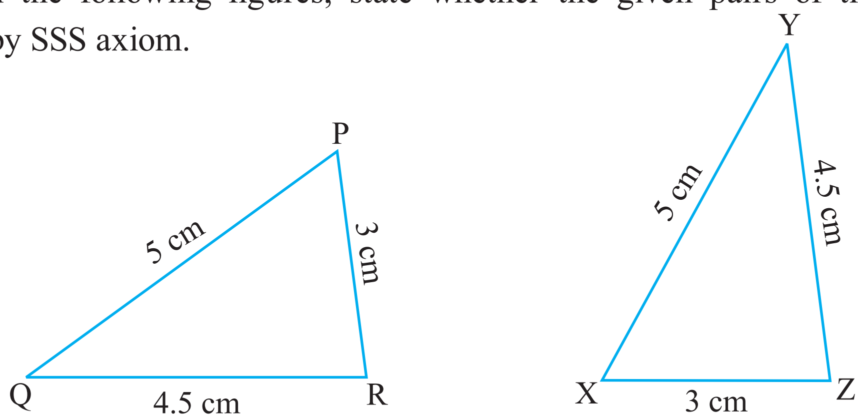
Here, we observe that $AB = PQ$, $BC = QR$, $CA = RP$.

It can be written as $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = 1$.

What will happen when the ratio is not equal to 1?

Example 3.7

From the following figures, state whether the given pairs of triangles are congruent by SSS axiom.



Solution

Compare the sides of the ΔPQR and ΔXYZ

$PQ = XY = 5\,cm$, $QR = YZ = 4.5\,cm$ and $RP = ZX = 3\,cm$.

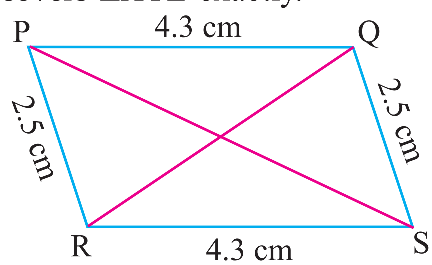
If we superpose $\Delta\ PQR$ on $\Delta\ XYZ$.

P lies on X, Q lies on Y, R lies on Z and ΔPQR covers ΔXYZ exactly.

$\therefore \Delta\ PQR \equiv \Delta XYZ$ [by SSS axiom].

Example 3.8

In the figure, PQSR is a parallelogram. $PQ = 4.3\,cm$ and $QR = 2.5\,cm$. Is $\Delta PQR \equiv \Delta PSR$?



Solution

Consider ΔPQR and ΔPSR. Here, $PQ = SR = 4.3\,cm$

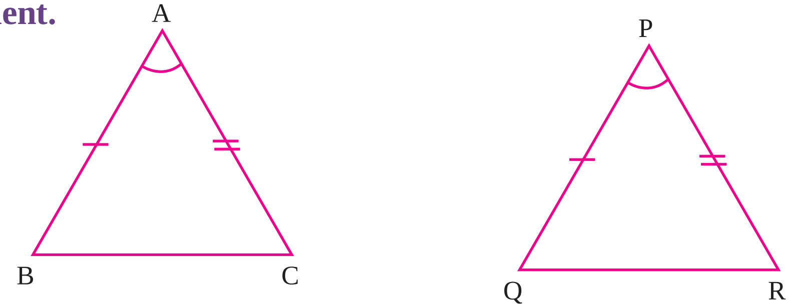
and $PR = QS = 2.5\,cm$. $PR = PR$ [common side]

$\therefore \Delta PQR \equiv \Delta RSP$ [by SSS axiom]

$\therefore \Delta PQR \not\equiv \Delta PSR$ [ΔRSP and ΔPSR are of different order]

(ii) SAS Axiom (Side-Angle-Side Axiom)

If any two sides and the included angle of a triangle are respectively equal to any two sides and the included angle of another triangle then the two triangles are congruent.



We consider two triangles, ΔABC and ΔPQR such that AB = PQ, AC = PR and included angle BAC = included angle QPR.

We superpose the trace copy of ΔABC on ΔPQR with AB along PQ and AC along PR.

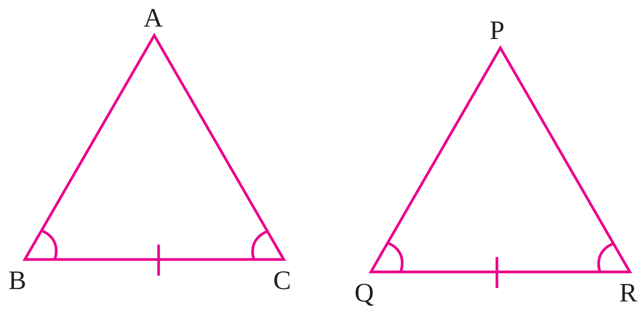
Now, A lies on P and B lies on Q and C lies on R. Since, AB = PQ and AC = PR, B lies on Q and C lies on R. BC covers QR exactly.

$\therefore \Delta ABC$ covers ΔPQR exactly.

Hence, $\Delta ABC \equiv \Delta PQR$

(iii) ASA Axiom (Angle-Side-Angle Axiom)

If two angles and a side of one triangle are respectively equal to two angles and the corresponding side of another triangle then the two triangles are congruent.



Consider the triangles, ΔABC and ΔPQR .

Here,

$BC = QR, \angle B = \angle Q, \angle C = \angle R$.

By the method of superposition, it is understood that $\angle ABC$ covers $\angle PQR$ exactly and $\angle BCA$ covers $\angle QRP$ exactly.

So, B lies on Q and C lies on R.

Hence A lies on P. $\therefore \Delta ABC$ covers ΔPQR exactly. Hence, $\Delta ABC \equiv \Delta PQR$.

As the triangles are congruent, we get remaining corresponding parts are also equal.

(i.e.) AB = PQ, AC = PR and $\angle A = \angle P$

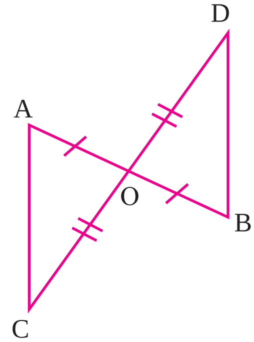
Activity

Prove the following axioms using the paper cuttings
a) SSS Axiom and b) ASA Axiom

Representation: The Corresponding Parts of Congruence Triangles are Congruent is represented in short form as c.p.c.t.c. Hereafter this notation will be used in the problems.

Example 3.9

AB and CD bisect each other at O. Prove that AC = BD.



Solution

Given : O is mid point of AB and CD.

$\therefore$ AO = OB and CO = OD

To prove : AC = BD

Proof : Consider ΔAOC and ΔBOD

AO = OB [Given]

CO = OD [Given]

$\angle$AOC = $\angle$BOD [Vertically Opposite angle]

ΔAOC $\equiv$ ΔBOD [by SAS axiom]

Hence we get, AC = BD [by c.p.c.t.c.]

Example 3.10

In the given figure, ΔDAB and ΔCAB are on the same base AB. Prove that ΔDAB $\equiv$ ΔCAB

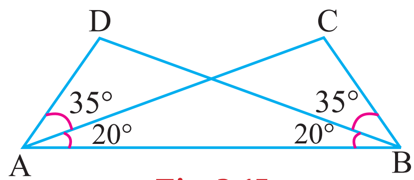


Fig. 3.15

Solution

Consider ΔDAB and ΔCAB

$\angle$DAB = $35^\circ + 20^\circ = 55^\circ$ = $\angle$CBA [Given]

$\angle$DBA = $\angle$CAB = 20° [Given]

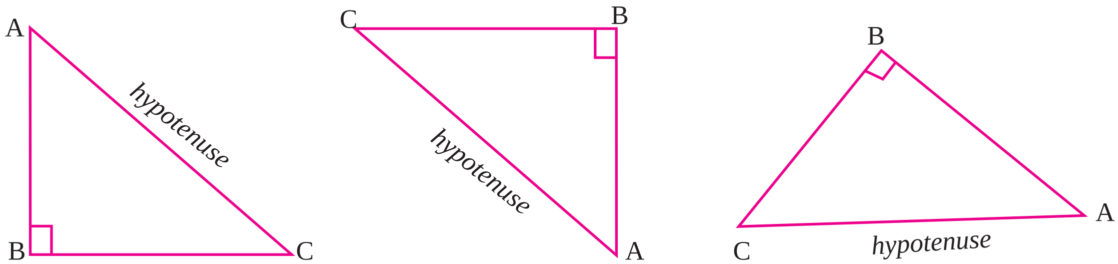
AB is common to both the triangles.

$\therefore$ ΔDBA $\equiv$ ΔCAB [by ASA axiom]

Hypotenuse

Do you know what is meant by hypotenuse ?

Hypotenuse is a word related with right angled triangle.



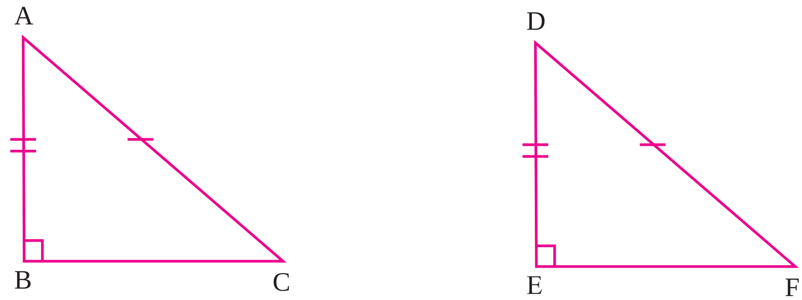
Consider the right angled triangle ABC. $\angle$B is a right angle.

The side opposite to right angle is known as the hypotenuse.

Here AC is hypotenuse.

(iv) RHS Axiom (Right angle - Hypotenuse - Side)

If the hypotenuse and one side of the right angled triangle are respectively equal to the hypotenuse and a side of another right angled triangle, then the two triangles are congruent.



Consider ΔABC and ΔDEF where, $\angle B = \angle E = 90^\circ$

Hypotenuse AC = Hypotenuse DF [Given]

Side AB = Side DE [Given]

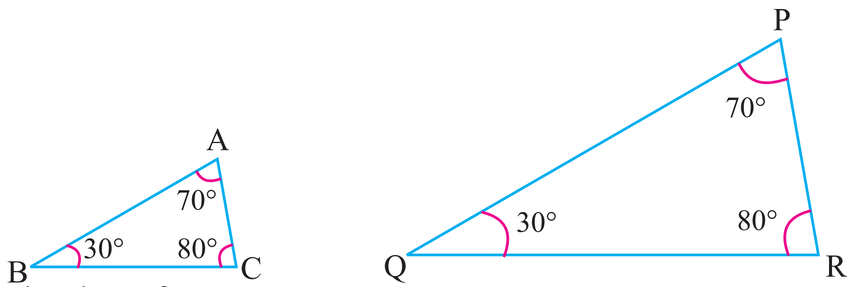
By the method of superposing, we see that $\Delta ABC \equiv \Delta DEF$.

3.3.4 Conditions which are not sufficient for congruence of triangles

(i) AAA (Angle - Angle - Angle)

It is not a sufficient condition for congruence of triangle. Why?

Let us find out the reason. Consider the following triangles.



In the above figures,

$$\angle A = \angle P, \angle B = \angle Q \text{ and } \angle C = \angle R$$

But size of ΔABC is smaller than the size of ΔPQR.

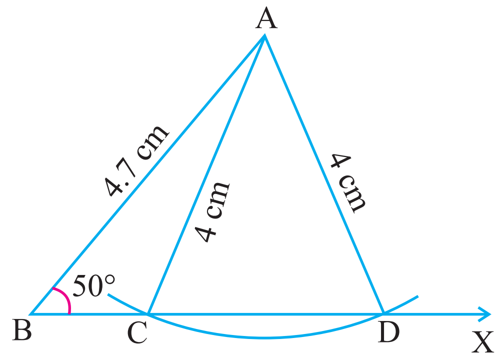
$\therefore$ When ΔABC is superposed on the ΔPQR, they will not cover each other exactly. $\therefore \Delta ABC \not\equiv \Delta PQR$.

(ii) SSA (Side-Side-Angle)

We can analyse a case as follows:

Construct ΔABC with the measurements $\angle B = 50^\circ$, AB = 4.7 cm and AC = 4 cm. Produce BC to X. With A as centre and AC as radius draw an arc of 4 cm. It will cut BX at C and D.

$\therefore$ AD is also 4cm [$\because$ AC and AD are the radius of the same circle]



Consider ΔABC and ΔABD.

$\angle B$ is common.

AB is common and AC = AD = 4cm [by construction]

Side AC, side AB and $\angle B$ of ΔABC and side AD , side AB and $\angle B$ of ΔABD are respectively congruent to each others. But BC and BD are not equal.

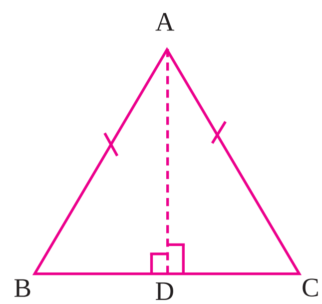
$\therefore \Delta ABC \not\equiv \Delta ABD$.

Example 3.11

Prove that the angles opposite to equal side of a triangle are equal.

Solution

ABC is a given triangle with, AB = AC.



To prove : Angle opposite to AB = Angle opposite to AC (i.e.) $\angle C = \angle B$.

Construction : Draw AD perpendicular to BC.

$\therefore \angle ADB = \angle ADC = 90^\circ$

Proof :

Condiser ΔABD and ΔACD.

AD is common

	$AB = AC$		[ΔABC is an isosecles]
	$\angle ADB = \angle ADC = 90^\circ$		[by construction]
	$\therefore \Delta ADB \equiv \Delta ADC$		[by RHS axiom]
Hence	$\angle ABD = \angle ACD$		[by c.p.c.t.c]
	(or) $\angle ABC = \angle ACB$.		
	$\angle B = \angle C$.	Hence the proof.	

This is known as Isosceles triangle theorem.

Example 3.12

Prove that the sides opposite to equal angles of a triangle are equal.

Solution

Given : In a ΔABC, $\angle B = \angle C$.

To prove : AB = AC.

Construction : Draw AD perpendicular to BC.

Proof :

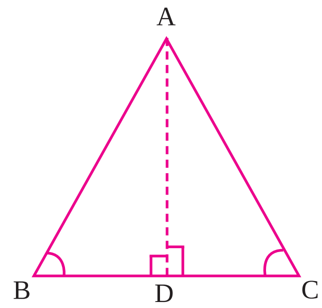
$\angle ADB = \angle ADC = 90°$ [by construction]

$\angle B = \angle C$ [given]

AD is common side.

$\therefore \Delta ADB \equiv \Delta ADC$ (by AAS axiom)

Hence, $AB = AC.$ [by c.p.c.t.c]



So, the sides opposite to equal angles of a triangle are equal.

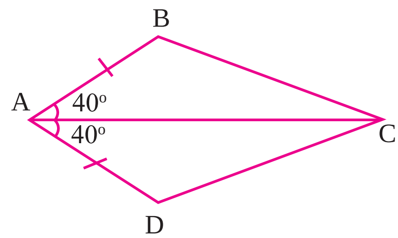
This is the converse of Isosceles triangle theorem.

Example 3.13

In the given figure $AB = AD$ and $\angle BAC = \angle DAC$. Is $\Delta ABC \equiv \Delta ADC$?

If so, state the other pairs of corresponding parts.

Solution



In ΔABC and ΔADC, AC is common.

$\angle BAC = \angle DAC$ [given]

$AB = AD$ [given]

$\therefore \Delta ABC \equiv \Delta ADC$ [by SAS axiom]

So, the remaining pairs of corresponding parts are

$BC = DC$, $\angle ABC = \angle ADC$, $\angle ACB = \angle ACD$. [by c.p.c.t.c]

Example 3.14

ΔPQR is an isosceles triangle with $PQ = PR$, QP is produced to S and PT bisects the extension angle $2x°$. Prove that $\angle Q = x°$ and hence prove that $PT \parallel QR$.

Solution

Given : ΔPQR is an isosceles triangle with $PQ = PR$.

Proof : PT bisects exterior angle $\angle SPR$ and therefore $\angle SPT = \angle TPR = x°$.

$\therefore \angle Q = \angle R$. [Property of an isosceles triangle]

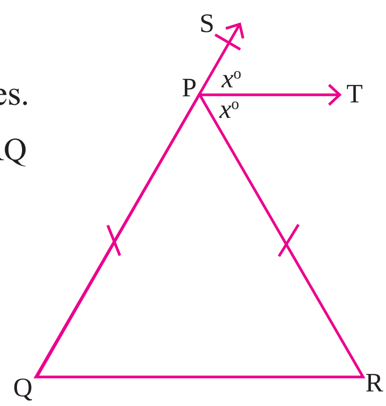
Also we know that in any triangle,

exterior angle = sum of the interior opposite angles.

$\therefore$ In ΔPQR, Exterior angle $\angle SPR = \angle PQR + \angle PRQ$

$$2x° = \angle Q + \angle R$$
$$= \angle Q + \angle Q$$
$$2x° = 2\angle Q$$
$$x° = \angle Q$$

Hence $\angle Q = x°$.



To prove : PT ∥ QR

Lines PT and QR are cut by the transversal SQ. We have $\angle SPT = x°$.

We already proved that $\angle Q = x°$.

Hence, $\angle SPT$ and $\angle PQR$ are corresponding angles.$\therefore$ PT ∥ QR.

EXERCISE 3.2

1. Choose the correct answer:

(i) In the isosceles ΔXYZ, given XY = YZ then which of the following angles are equal?

(A) $\angle X$ and $\angle Y$ (B) $\angle Y$ and $\angle Z$ (C) $\angle Z$ and $\angle X$ (D) $\angle X$, $\angle Y$ and $\angle Z$

(ii) In ΔABC and ΔDEF, $\angle B = \angle E$, AB = DE, BC = EF. The two triangles are congruent under ______ axiom

(A) SSS (B) AAA (C) SAS (D) ASA

(iii) Two plane figures are said to be congruent if they have

(A) the same size (B) the same shape

(C) the same size and the same shape (D) the same size but not same shape

(iv) In a triangle ABC, $\angle A = 40°$ and AB = AC, then ABC is ______ triangle.

(A) a right angled (B) an equilateral (C) an isosceles (D) a scalene

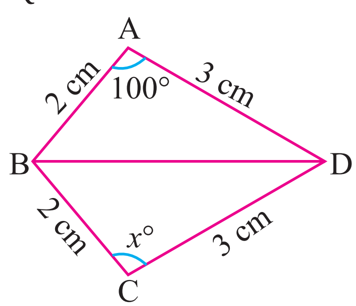
(v) In the triangle ABC, when $\angle A = 90°$ the hypotenuse is ------

(A) AB (B) BC (C) CA (D) None of these

(vi) In the ΔPQR the angle included by the sides PQ and PR is

(A) $\angle P$ (B) $\angle Q$

(C) $\angle R$ (D) None of these

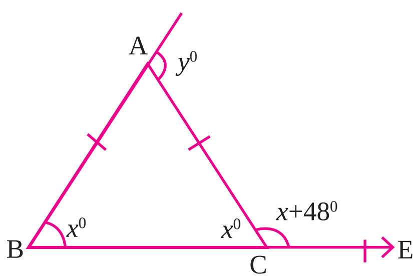


(vii) In the figure, the value of $x°$ is ----------

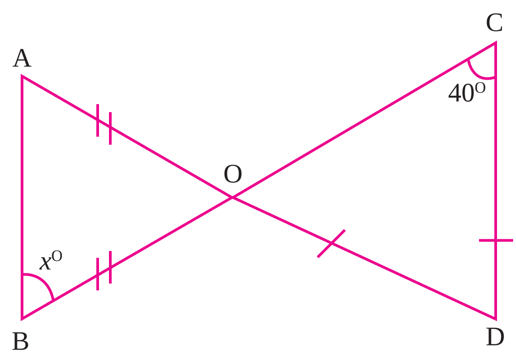
(A) 80° (B) 100°

(C) 120° (D) 200°

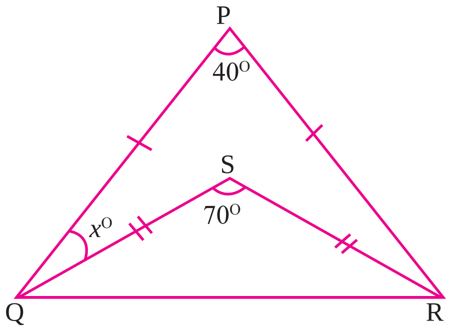
2. In the figure, ABC is a triangle in which AB = AC. Find $x°$ and $y°$.



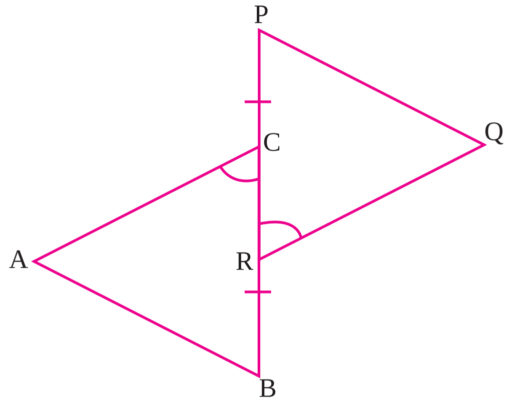
3. In the figure, Find $x°$.



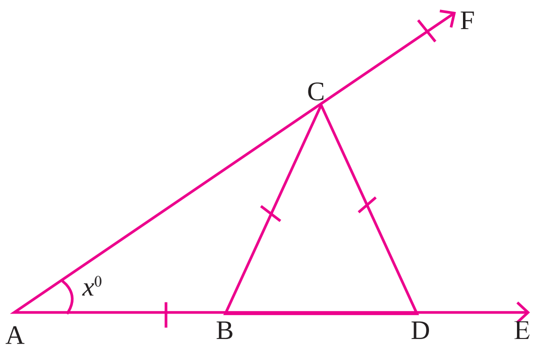
4. In the figure ΔPQR and ΔSQR are isosceles triangles. Find $x°$.



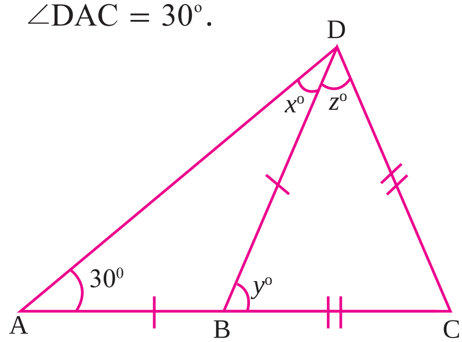
5. In the figure, it is given that BR = PC and $\angle ACB = \angle QRP$ and $AB \parallel PQ$. Prove that AC = QR.



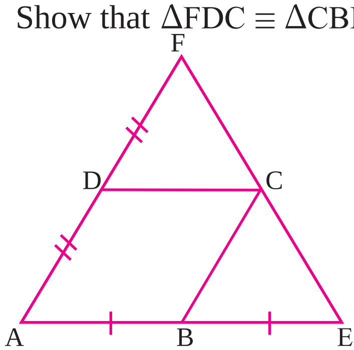
6. In the figure, AB = BC = CD , $\angle A = x°$. Prove that $\angle DCF = 3\angle A$.



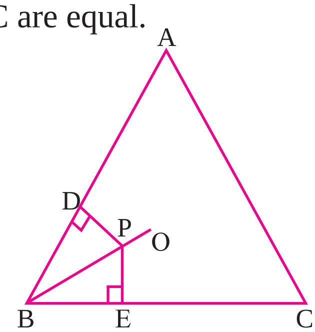
7. Find $x°$, $y°$, $z°$ from the figure, where AB = BD, BC = DC and $\angle DAC = 30°$.



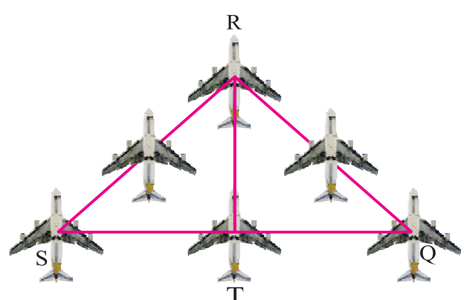
8. In the figure, ABCD is a parallelogram. AB is produced to E such that AB = BE. AD produced to F such that AD = DF. Show that $\Delta FDC \equiv \Delta CBE$.



9. In figure, BO bisects $\angle ABC$ of ΔABC. P is any point on BO. Prove that the perpendicular drawn from P to BA and BC are equal.



10. The Indian Navy flights fly in a formation that can be viewed as two triangles with common side. Prove that $\Delta SRT \equiv \Delta QRT$, if T is the midpoint of SQ and SR = RQ.



Concept Summary

- The sum of the three angles of a triangle is 180°.
- If the sides of a triangle is produced, the exterior angle so formed, is equal to the sum of the two interior opposite angles.
- Any two sides of a triangle together is greater than the third side.
- Two plane figures are Congruent if each when superposed on the other covers it exactly. It is denoted by the symbol "$\equiv$".
- Two triangles are said to be congruent, if three sides and the three angles of one triangle are respectively equal to three sides and three angles of the other.
- **SSS Axiom:** If three sides of a triangle are respectively equal to the three sides of another triangle then the two triangles are congruent.
- **SAS Axiom:** If any two sides and the included angle of a triangle are respectively equal to any two sides and the included angle of another triangle then the two triangles are congruent.
- **ASA Axiom:** If two angles and a side of one triangle are respectively equal to two angles and the corresponding side of another triangle then the two triangles are congruent.
- **RHS Axiom:** If the hypotenuse and one side of the right angled triangle are respectively equal to the hypotenuse and a side of another right angled triangle, then the two triangles are congruent.

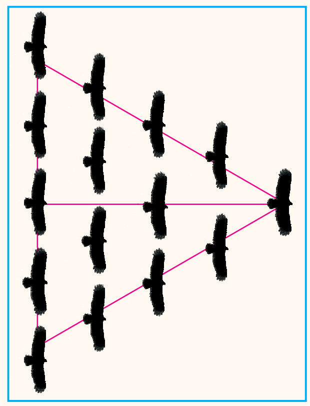
Mathematic Club Activity

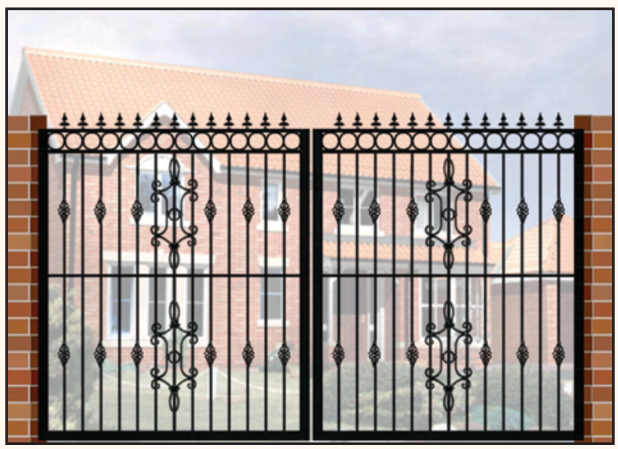
The Importance of Congruency

In our daily life, we use the concept of congruence in many ways. In our home, we use double doors which is congruent to each other. Mostly our house double gate is congruent to each other. The wings of birds are congruent to each other. The human body parts like hands, legs are congruent to each other. We can say many examples like this.

Birds while flying in the sky, they fly in the formation of a triangle. If you draw a median through the leading bird you can see a congruence. If the congruency collapses then the birds following at the end could not fly because they losses their stability.

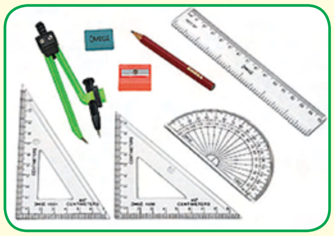
Now, try to identify the congruence structures in the nature and in your practical life.





Practical Geometry

4



4.1 Introduction

Ancient Egyptians demonstrated practical knowledge of geometry through surveying and construction of projects. Ancient Greeks practised experimental geometry in their culture. They have performed variety of constructions using ruler and compass.

Geometry is one of the earliest branches of Mathematics. Geometry can be broadly classified into Theoretical Geometry and Practical Geometry. Theoretical Geometry deals with the principles of geometry by explaining the construction of figures using rough sketches. Practical Geometry deals with constructing of exact figures using geometrical instruments.

We have already learnt in the previous classes, the definition, properties and formulae for the area of some plane geometrical figures. In this chapter let us learn to construct some specific plane geometrical figures.

Gauss

[1777-1855 A.D.]

Gauss was a German Mathematician. At the age of seventeen Gauss investigated the constructibility of regular '***p*-gons**' (polygons with p-sides) where p is prime number. The construction was then known only for $p = 3$ and $p = 5$. Gauss discovered that the regular p-gon is constructible if and only if p is prime "Fermat Number" (i.e.) $p = 2^{2^n} + 1$

4.2 Quadrilateral

4.2.1 Introduction

We have learnt in VII standard about quadrilateral and properties of quadrilateral. Let us recall them.

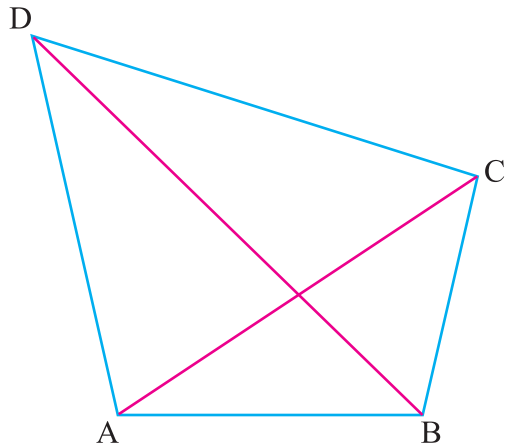


Fig. 4.1

In Fig. 4.1, A, B, C, D are four points in a plane. No three points lie on a line.

$\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$ intersect only at the vertices. We have learnt that quadrilateral is a four sided plane figure. We know that the sum of measures of the four angles of a quadrilateral is 360°.

$(\overline{AB}, \overline{AD})$, $(\overline{AB}, \overline{BC})$, $(\overline{BC}, \overline{CD})$, $(\overline{CD}, \overline{DA})$ are adjacent sides. $\overline{AC}$ and $\overline{BD}$ are the diagonals.

$\angle A$, $\angle B$, $\angle C$ and $\angle D$ (or $\angle DAB$, $\angle ABC$, $\angle BCD$, $\angle CDA$) are the angles of the quadrilateral ABCD.

$$\therefore\ \angle A + \angle B + \angle C + \angle D = 360°$$

Note : (i) We should name the quadrilateral in cyclic ways such as ABCD and BCDA.

(ii) Square, Rectangle, Rhombus, Parallelogram, Trapezium are all **Quadrilaterals.**

(iii) A quadrilateral has four vertices, four sides, four angles and two diagonals.

4.2.2 Area of a Quadrilateral

Let ABCD be any quadrilateral with $\overline{BD}$ as one of its diagonals.

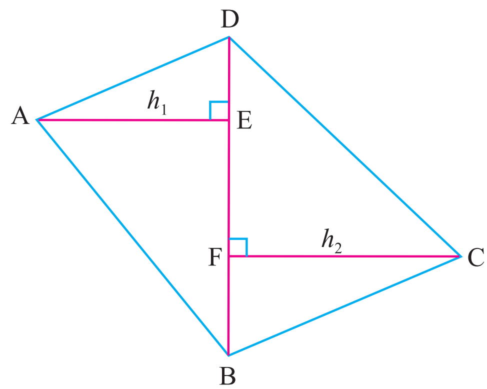


Fig. 4.2

Let $\overline{AE}$ and $\overline{FC}$ be the perpendiculars drawn from the vertices A and C on diagonal $\overline{BD}$.

From the Fig. 4.2

Area of the quadrilateral ABCD

$$= \text{Area of } \triangle\ ABD + \text{Area of } \triangle\ BCD$$

$$= \frac{1}{2} \times BD \times AE + \frac{1}{2} \times BD \times CF$$

$$= \frac{1}{2} \times BD \times (AE + CF) = \frac{1}{2} \times d \times (h_1 + h_2) \text{ sq. units.}$$

where BD = d, AE = h_1 and CF = h_2.

Area of a quadrilateral is half of the product of a diagonal and the sum of the altitudes drawn to it from its opposite vertices. That is,

$A = \frac{1}{2} d (h_1 + h_2)$ sq. units, where 'd' is the diagonal; 'h_1' and 'h_2' are the altitudes drawn to the diagonal from its opposite vertices.

Activity

By using paper folding technique, verify $A = \frac{1}{2} d (h_1 + h_2)$

4.2.3 Construction of a Quadrilateral

In this class, let us learn how to construct a quadrilateral.

To construct a **quadrilateral** first we construct a triangle from the given data. Then, we find the fourth vertex.

To construct a triangle, we require three independent measurements. Also we need two more measurements to find the fourth vertex. Hence, we need **five independent** measurements to construct a quadrilateral.

We can construct, a quadrilateral, when the following measurements are given:

- **(i) Four sides and one diagonal**
- **(ii) Four sides and one angle**
- **(iii) Three sides, one diagonal and one angle**
- **(iv) Three sides and two angles**
- **(v) Two sides and three angles**

4.2.4 Construction of a quadrilateral when four sides and one diagonal are given

Example 4.1

Construct a quadrilateral ABCD with AB = 4 cm, BC = 6 cm, CD = 5.6 cm DA = 5 cm and AC = 8 cm. Find also its area.

Solution

Given: AB = 4 cm, BC = 6 cm, CD = 5.6 cm
DA = 5 cm and AC = 8 cm.

Rough Diagram

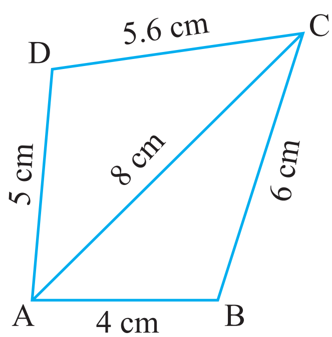


Fig. 4.3

To construct a quadrilateral

Steps for construction

Step 1 : Draw a rough figure and mark the given measurements.

Step 2 : Draw a line segment AB = 4 cm.

Step 3 : With A and B as centres draw arcs of radii 8 cm and 6 cm respectively and let them cut at C.

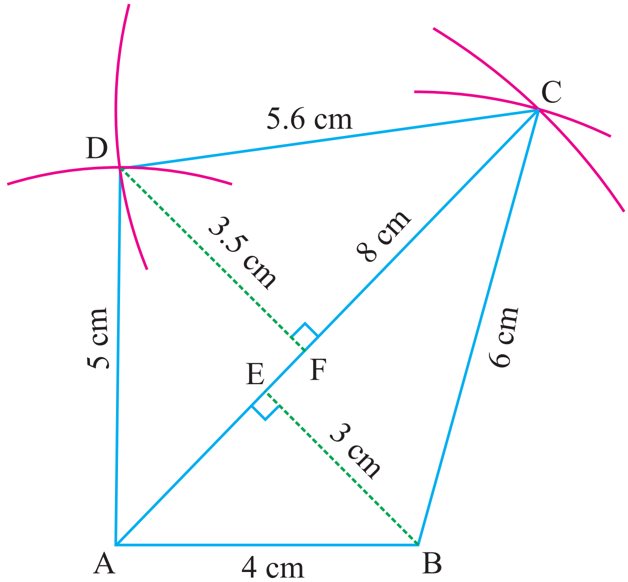


Fig. 4.4

Step 4 : Join $\overline{AC}$ and $\overline{BC}$.

Step 5 : With A and C as centres draw arcs of radii 5 cm, and 5.6 cm respectively and let them cut at D.

Step 6 : Join $\overline{AD}$ and $\overline{CD}$.

ABCD is the required quadrilateral.

Step 7 : From B draw $\overline{BE} \perp \overline{AC}$ and from D draw $\overline{DF} \perp \overline{AC}$, then measure the lengths of BE and DF. BE = $h_1 = 3$ cm and DF = $h_2 = 3.5$ cm. AC = d = 8 cm.

Calculation of area:

In the quadrilateral ABCD, $d =$ 8 cm, $h_1 = 3$ cm and $h_2 = 3.5$ cm.

Area of the quadrilateral ABCD $= \frac{1}{2}\, d\,(h_1 + h_2)$

$= \frac{1}{2}(8)(3 + 3.5)$

$= \frac{1}{2} \times 8 \times 6.5$

= 26 cm^2.

4.2.5 Construction of a quadrilateral when four sides and one angle are given

Example 4.2

Construct a quadrilateral ABCD with AB = 6 cm, BC = 4 cm, CD = 5 cm, DA = 4.5 cm, $\angle ABC = 100°$ and find its area.

Solution

Given:

AB = 6 cm, BC = 4 cm,CD = 5 cm, DA = 4.5 cm $\angle ABC = 100°$.

To construct a quadrilateral

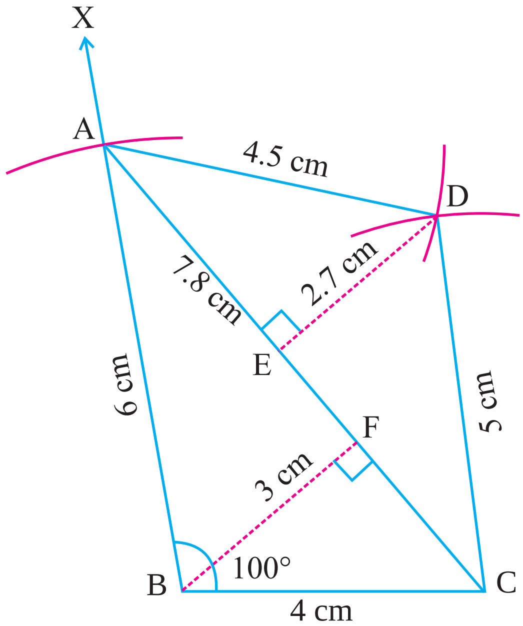


Fig.4.6

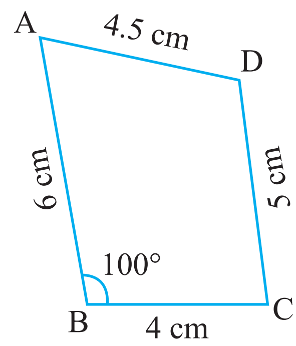


Fig. 4.5

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurments.

Step 2 : Draw a line segment BC = 4 cm.

Step 3 : At B on $\overline{BC}$ make $\angle CBX$ whose measure is 100°.

Step 4 : With B as centre and radius 6 cm draw an arc. This cuts $\overrightarrow{BX}$ at A. Join $\overline{CA}$

Step 5 : With C and A as centres, draw arcs of radii 5 cm and 4.5 cm respectively and let them cut at D.

Step 6 : Join $\overline{CD}$ and $\overline{AD}$.

ABCD is the required quadrilateral.

Step 7 : From B draw $\overline{BF} \perp \overline{AC}$ and from D draw $\overline{DE} \perp \overline{AC}$. Measure the lengths of BF and DE. BF = $h_1 = 3$ cm, DE = $h_2 = 2.7$ cm and AC = d = 7.8 cm.

Calculation of area:

In the quadrilateral ABCD, d = 7.8 cm, h_1 = 3 cm and h_2 = 2.7 cm.

Area of the quadrilateral ABCD $= \frac{1}{2} d (h_1 + h_2)$

$= \frac{1}{2}(7.8)(3 + 2.7)$

$= \frac{1}{2} \times 7.8 \times 5.7 = 22.23 \text{ cm}^2$.

MATHEMATICS

4.2.6 Construction of a quadrilateral when three sides, one diagonal and one angle are given

Example 4.3

Construct a quadrilateral PQRS with PQ = 4 cm, QR = 6 cm, PR = 7 cm, PS = 5 cm and $\angle PQS = 40°$ and find its area.

Solution

Given: PQ = 4 cm, QR = 6 cm, PR= 7 cm,
PS= 5 cm and $\angle PQS = 40°$.

Rough Diagram

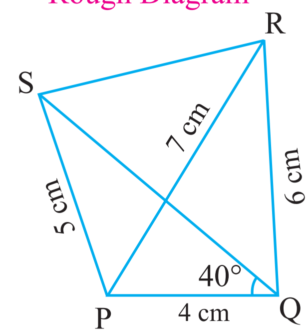


Fig. 4.7

To construct a quadrilateral

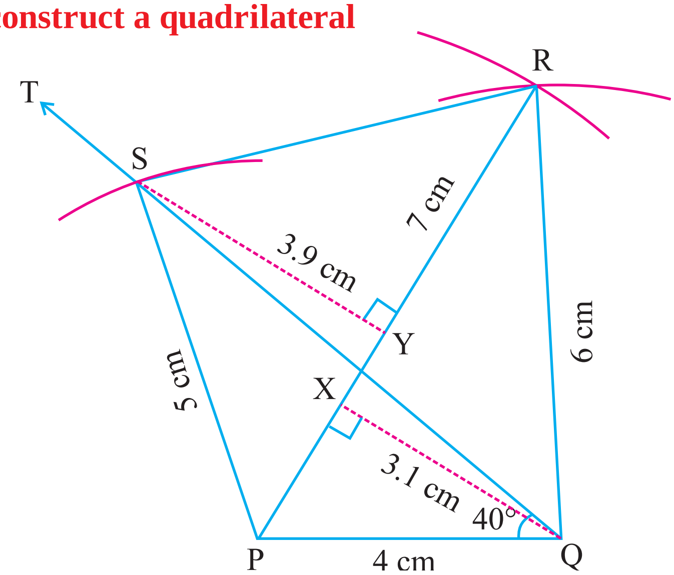


Fig. 4.8

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment PQ = 4 cm.

Step 3 : With P and Q as centres draw arcs of radii 7 cm and 6 cm respectively and let them cut at R.

Step 4 : Join $\overline{PR}$ and $\overline{QR}$.

Step 5 : At Q on $\overline{PQ}$ make $\angle PQT$ whose measure is 40°.

Step 6 : With P as centre and radius 5 cm draw an arc. This cuts $\overrightarrow{QT}$ at S.

Step 7 : Join $\overline{PS}$.

PQRS is the required quadrilateral.

Step 8 : From Q draw $\overline{QX} \perp \overline{PR}$ and from S draw $\overline{SY} \perp \overline{PR}$. Measure the lengths QX and SY. QX = $h_1 = 3.1$ cm, SY = $h_2 = 3.9$ cm.
PR = d = 7 cm.

Calculation of area:

In the quadrilateral PQRS, $d = 7$ cm, $h_1 = 3.1$ cm and $h_2 = 3.9$ cm.

$$\text{Area of the quadrilateral PQRS} = \frac{1}{2}\, d\,(h_1 + h_2)$$
$$= \frac{1}{2}(7)(3.1 + 3.9)$$
$$= \frac{1}{2} \times 7 \times 7$$
$$= 24.5 \text{ cm}^2.$$

4.2.7 Construction of a quadrilateral when three sides and two angles are given

Example 4.4

Construct a quadrilateral ABCD with AB = 6.5 cm, AD = 5 cm, CD = 5 cm, $\angle BAC = 40°$ and $\angle ABC = 50°$, and also find its area.

Solution

Given:

AB = 6.5 cm, AD = 5 cm, CD = 5 cm,

$\angle BAC = 40°$ and $\angle ABC = 50°$.

Rough Diagram

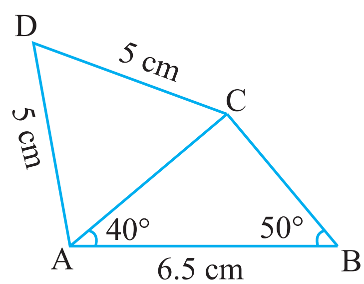


Fig. 4.9

To construct a quadrilateral

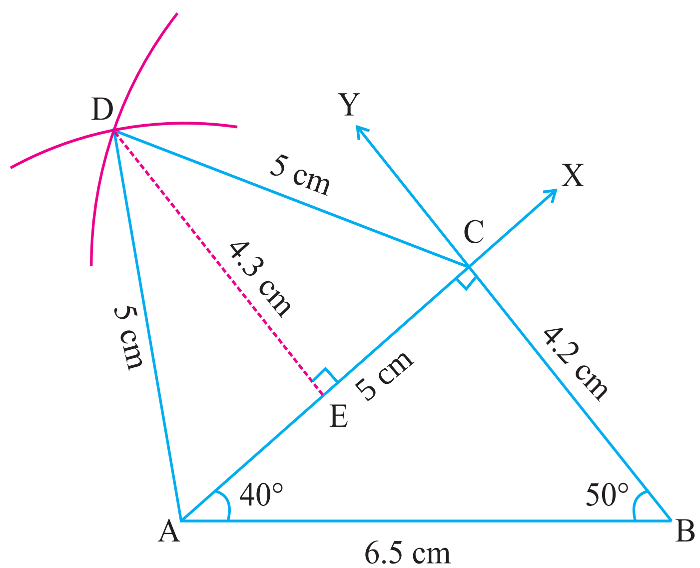


Fig. 4.10

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AB = 6.5 cm.

Step 3 : At A on $\overline{AB}$ make $\angle BAX$ whose measure is 40° and at B on $\overline{AB}$ make $\angle ABY$ whose measure is 50°. They meet at C.

Step 4 : With A and C as centres draw arcs of radius 5 cm and 5 cm respectively and let them cut at D.

Step 5 : Join $\overline{AD}$ and $\overline{CD}$.

ABCD is the required quadrilateral.

Step 6 : From D draw $\overline{DE} \perp \overline{AC}$ and from B draw $\overline{BC} \perp \overline{AC}$. Then measure the lengths of BC and DE. BC = h_1 = 4.2 cm, DE = h_2 = 4.3 cm and AC = d = 5 cm.

Calculation of area:

In the quadrilateral ABCD, d = 5 cm, BC = h_1 = 4.2 cm and h_2 = 4.3 cm.

$$\text{Area of the quadrilateral ABCD} = \frac{1}{2} d\,(h_1 + h_2)$$

$$= \frac{1}{2}(5)(4.2 + 4.3)$$

$$= \frac{1}{2} \times 5 \times 8.5 = 21.25 \text{ cm}^2.$$

4.2.8 Construction of a quadrilateral when two sides and three angles are given

Example 4.5

Construct a quadrilateral ABCD with AB = 6 cm, AD = 6 cm, $\angle$ABD = 45°, $\angle$BDC = 40° and $\angle$DBC = 40°. Find also its area.

Solution

Given: AB = 6 cm, AD = 6 cm, $\angle$ABD = 45°, $\angle$BDC = 40° and $\angle$DBC = 40°.

Rough Diagram

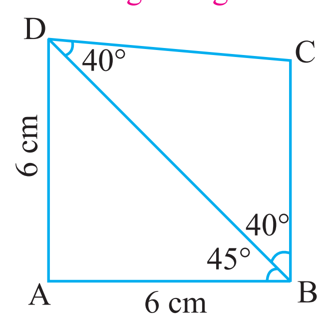


Fig. 4.11

To construct a quadrilateral

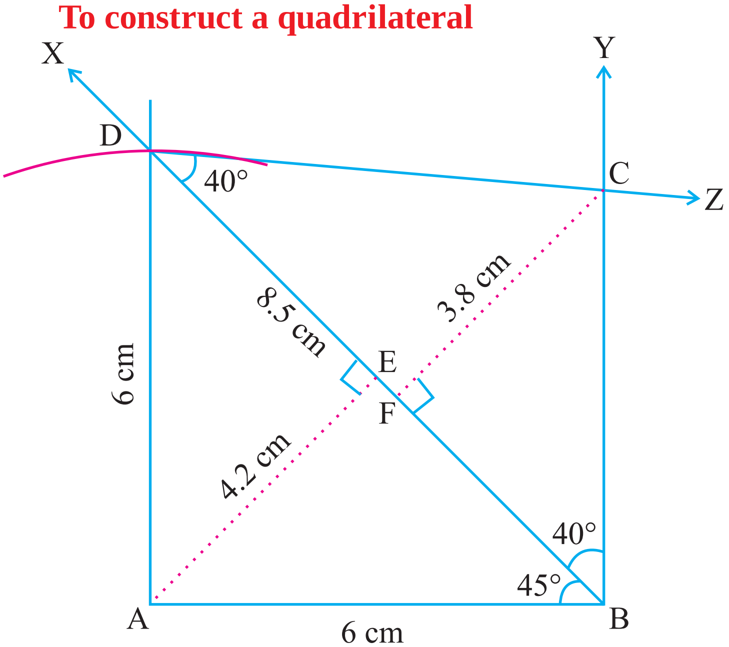


Fig. 4.12

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AB = 6 cm.

Step 3 : At B on $\overline{AB}$ make $\angle ABX$ whose measure is 45°.

Step 4 : With A as centre and 6 cm as radius draw an arc. Let it cut $\overrightarrow{BX}$ at D.

Step 5 : Join $\overline{AD}$.

Step 6 : At B on $\overline{BD}$ make $\angle DBY$ whose measure is 40°.

Step 7 : At D on $\overline{BD}$ make $\angle BDZ$ whose measure is 40°.

Step 8 : Let $\overrightarrow{BY}$ and $\overrightarrow{DZ}$ intersect at C.

ABCD is the required quadrilateral.

Step 9 : From A draw $\overline{AE} \perp \overline{BD}$ and from C draw $\overline{CF} \perp \overline{BD}$. Then measure the lengths of AE and CF. AE = h_1 = 4.2 cm, CF = h_2 = 3.8 cm and BD = d = 8.5 cm.

Calculation of area:

In the quadrilateral ABCD, d = 8.5 cm, h_1 = 4.2 cm and h_2 = 3.8 cm.

$$\text{Area of the quadrilateral ABCD} = \frac{1}{2} d\,(h_1 + h_2)$$

$$= \frac{1}{2}(8.5)\,(4.2 + 3.8)$$

$$= \frac{1}{2} \times 8.5 \times 8 = 34 \text{ cm}^2.$$

EXERCISE 4.1

Draw quadrilateral ABCD with the following measurements. Find also its area.

1. AB = 5 cm, BC = 6 cm, CD = 4 cm, DA= 5.5 cm and AC = 7 cm.
2. AB = 7 cm, BC = 6.5 cm, AC = 8 cm, CD= 6 cm and DA = 4.5 cm.
3. AB = 8 cm, BC = 6.8 cm, CD = 6 cm, AD= 6.4 cm and $\angle$ B = 50°.
4. AB = 6 cm, BC = 7 cm, AD = 6 cm, CD= 5 cm, and $\angle$ BAC = 45°.
5. AB = 5.5 cm, BC = 6.5 cm, BD = 7 cm, AD= 5 cm and $\angle$ BAC= 50°.
6. AB = 7 cm, BC = 5 cm, AC = 6 cm, CD= 4 cm, and $\angle$ ACD = 45°..
7. AB = 5.5 cm, BC = 4.5cm, AC = 6.5 cm, $\angle$ CAD = 80° and $\angle$ ACD = 40°.
8. AB = 5 cm, BD = 7 cm, BC = 4 cm, $\angle$ BAD = 100° and $\angle$ DBC = 60.
9. AB = 4 cm, AC = 8 cm, $\angle$ ABC = 100°, $\angle$ ABD = 50° and

   $\angle$ CAD = 40°.
10. AB = 6 cm, BC = 6 cm, $\angle$ BAC = 50°, $\angle$ ACD = 30° and $\angle$ CAD = 100°.

4.3 Trapezium

4.3.1 Introduction

In the class VII we have learnt special quadrilaterals such as trapezium and isosceles trapezium. We have also learnt their properties. Now we recall the definition of a trapezium.

A quadrilateral in which only one pair of opposite sides are parallel is called a trapezium.

4.3.2 Area of a trapezium

Let us consider the trapezium EASY

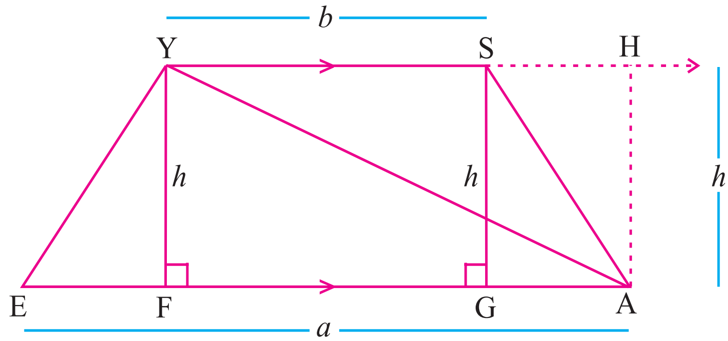


Fig. 4.13

We can partition the above trapezium into two triangles by drawing a diagonal $\overline{YA}$.

One triangle has base $\overline{EA}$ (EA = a units)

The other triangle has base $\overline{YS}$ (YS = b units)

We know $\overline{EA} \parallel \overline{YS}$

YF = HA = h units

Now, the area of $\triangle$ EAY is $\frac{1}{2}$ ah. The area of $\triangle$ YAS is $\frac{1}{2}$ bh.

Hence,

the area of trapezium EASY = Area of $\triangle$ EAY + Area of $\triangle$ YAS

$$= \frac{1}{2}ah + \frac{1}{2}bh$$

$$= \frac{1}{2}h(a+b) \text{ sq. units}$$

$$= \frac{1}{2} \times \text{height} \times (\text{Sum of the parallel sides}) \text{ sq. units}$$

Area of Trapezium

A = $\frac{1}{2}$ $h\ (a + b)$ sq. units where 'a' and 'b' are the lengths of the parallel sides and 'h' is the perpendicular distance between the parallel sides.

4.3.3 Construction of a trapezium

In general to construct a trapezium, we take the parallel sides which has greater measurement as base and on that base we construct a triangle with the given measurements such that the triangle lies between the parallel sides. Clearly the vertex opposite to the base of the triangle lies on the parallel side opposite to the base. We draw the line through this vertex parallel to the base. Clearly the fourth vertex lies on this line and this fourth vertex is fixed with the help of the remaining measurement. Then by joining the appropriate vertices we get the required trapezium.

To construct a trapezium we need four independent data.

We can construct a trapezium with the following given information:

- **(i) Three sides and one diagonal**
- **(ii) Three sides and one angle**
- **(iii) Two sides and two angles**
- **(iv) Four sides**

4.3.4 Construction of a trapezium when three sides and one diagonal are given

Example 4.6

Construct a trapezium ABCD in which $\overline{AB}$ is parallel to $\overline{DC}$, AB = 10 cm, BC = 5 cm, AC = 8 cm and CD = 6 cm. Find its area.

Solution

Given:

$\overline{AB}$ is parallel to $\overline{DC}$, AB = 10 cm,

BC = 5 cm, AC = 8 cm and CD = 6 cm.

Rough Diagram

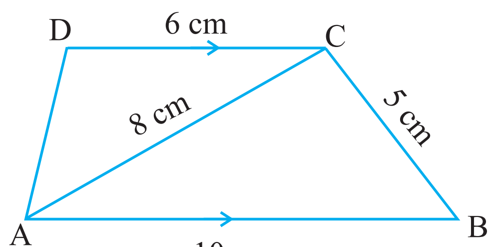


Fig. 4.14

To construct a trapezium

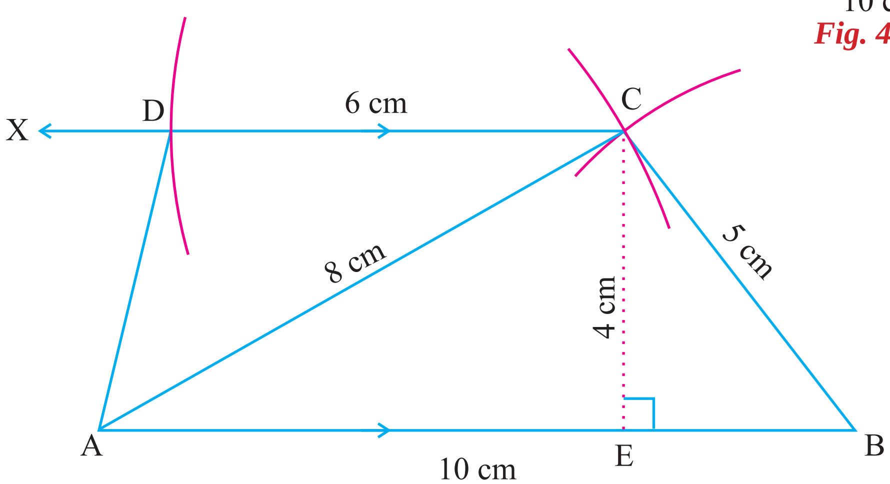


Fig. 4.15

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AB = 10 cm.

Step 3 : With A and B as centres draw arcs of radii 8 cm and 5 cm respectively and let them cut at C.

Step 4 : Join $\overline{AC}$ and $\overline{BC}$.

Step 5 : Draw $\overrightarrow{CX}$ parallel to $\overline{BA}$.

Step 6 : With C as centre and radius 6 cm draw an arc cutting $\overrightarrow{CX}$ at D.

Step 7 : Join $\overline{AD}$.

ABCD is the required trapezium.

Step 8 : From C draw $\overline{CE} \perp \overline{AB}$ and measure the length of CE.

CE = h = 4 cm.

AB = a = 10 cm, DC = b = 6 cm.

Calculation of area:

In the trapezium ABCD, a = 10 cm, b = 6 cm and h = 4 cm.

$$\text{Area of the trapezium ABCD} = \frac{1}{2}h(a+b)$$

$$= \frac{1}{2}(4)(10+6)$$

$$= \frac{1}{2} \times 4 \times 16$$

$$= 32 \text{ cm}^2.$$

4.3.5 Construction of a trapezium when three sides and one angle are given

Example 4.7

Construct a trapezium PQRS in which $\overline{PQ}$ is parallel to $\overline{SR}$, PQ = 8 cm $\angle$PQR = 70°, QR = 6 cm and PS = 6 cm. Calculate its area.

Solution

Given:

$\overline{PQ}$ is parallel to $\overline{SR}$, PQ = 8 cm, $\angle$PQR = 70°, QR = 6 cm and PS = 6 cm.

Rough Diagram

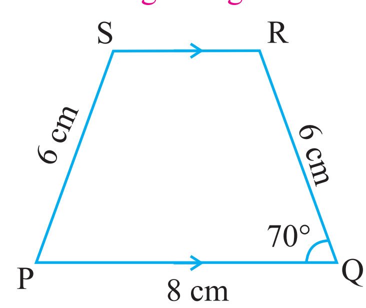


Fig 4.16

To construct a trapezium

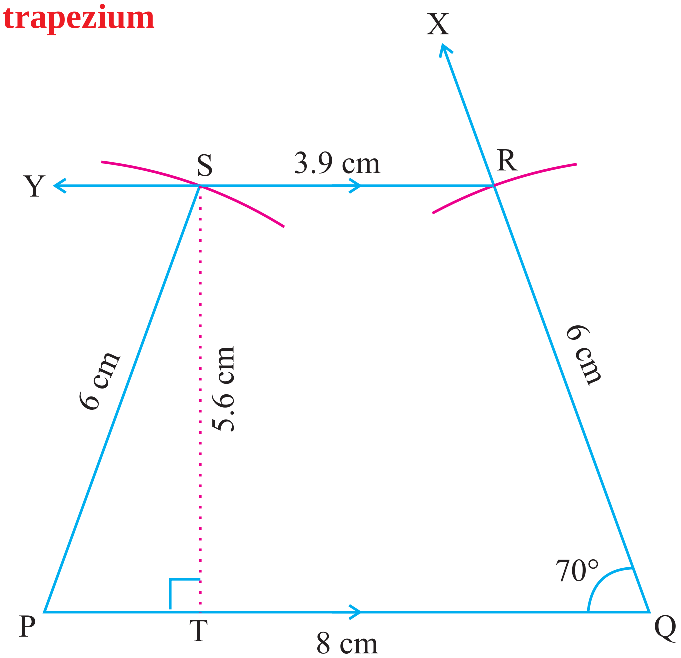


Fig. 4.17

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment PQ = 8 cm.

Step 3 : At Q on $\overline{PQ}$ make $\angle PQX$ whose measure is 70°.

Step 4 : With Q as centre and 6 cm as radius draw an arc. This cuts $\overrightarrow{QX}$ at R.

Step 5 : Draw $\overrightarrow{RY}$ parallel to $\overline{QP}$.

Step 6 : With P as centre and 6 cm as radius draw an arc cutting $\overrightarrow{RY}$ at S.

Step 7 : Join $\overline{PS}$.

PQRS is the required trapezium.

Step 8 : From S draw $\overline{ST} \perp \overline{PQ}$ and measure the length of ST.

ST = h = 5.6 cm,

RS = b = 3.9 cm. PQ = a = 8 cm.

Calculation of area:

In the trapezium PQRS, a = 8 cm, b = 3.9 cm and h = 5.6 cm.

Area of the trapezium PQRS $= \frac{1}{2} h (a + b)$

$= \frac{1}{2}(5.6)(8 + 3.9)$

$= \frac{1}{2} \times 5.6 \times 11.9$

$= 33.32 \text{ cm}^2$.

4.3.6. Construction of a trapezium when two sides and two angles are given

Example 4.8

Construct a trapezium ABCD in which $\overline{AB}$ is parallel to $\overline{DC}$, AB = 7 cm, BC = 6 cm, $\angle BAD = 80°$ and $\angle ABC = 70°$ and calculate its area.

Solution

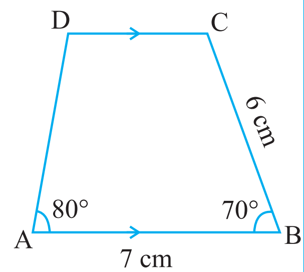


Fig. 4.18

Given:

$\overline{AB}$ is parallel to $\overline{DC}$, AB = 7 cm,

BC = 6 cm, $\angle BAD = 80°$ and $\angle ABC = 70°$.

To construct a trapezium

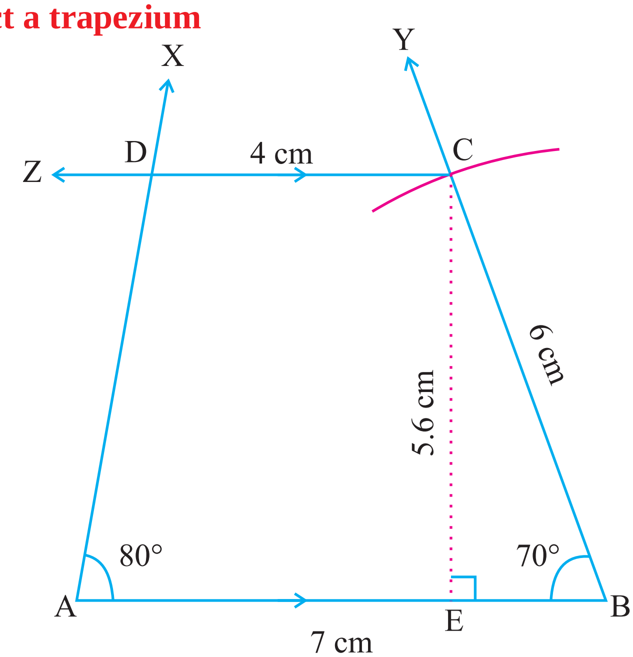


Fig. 4.19

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AB = 7 cm.

Step 3 : On $\overline{AB}$ at A make $\angle BAX$ measuring 80°.

Step 4 : On $\overline{AB}$ at B make $\angle ABY$ measuring 70°.

Step 5 : With B as centre and radius 6 cm draw an arc cutting $\overrightarrow{BY}$ at C.

Step 6 : Draw $\overrightarrow{CZ}$ parallel to $\overline{AB}$. This cuts $\overrightarrow{AX}$ at D.

ABCD is the required trapezium.

Step 7 : From C draw $\overline{CE} \perp \overline{AB}$ and measure the length of CE.

CE = h = 5.6 cm and CD = b = 4 cm.

Also, AB = a = 7 cm.

Calculation of area:

In the trapezium ABCD, $a = 7$ cm, $b = 4$ cm and $h = 5.6$ cm.

$$\text{Area of the trapezium ABCD} = \frac{1}{2}h(a+b)$$
$$= \frac{1}{2}(5.6)\ (7+4)$$
$$= \frac{1}{2}\times 5.6\times 11$$
$$= 30.8 \text{ cm}^2.$$

4.3.7. Construction of a trapezium when four sides are given

Example 4.9

Construct a trapezium ABCD in which $\overline{AB}$ is parallel to $\overline{DC}$, AB = 7 cm, BC = 5 cm, CD = 4 cm and AD = 5 cm and calculate its area.

Solution

Given:

$\overline{AB}$ is parallel to $\overline{DC}$, BC = 5 cm, CD = 4 cm and AD = 5 cm.

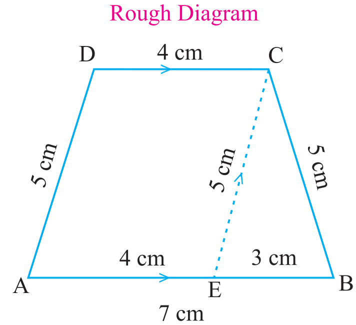


Fig. 4.20

To construct a trapezium

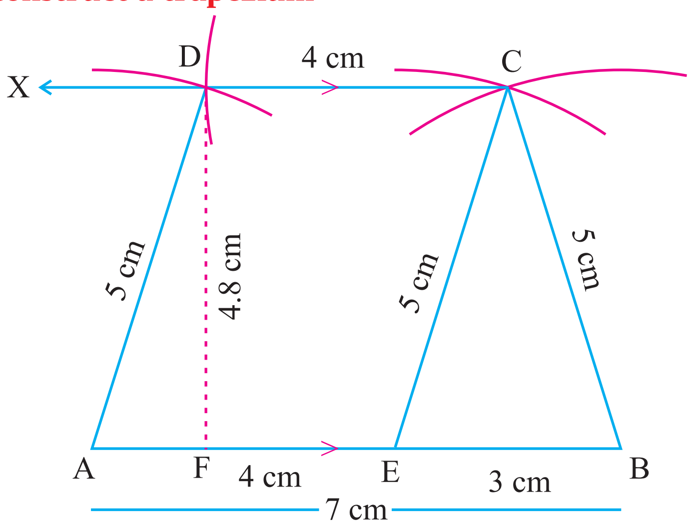


Fig. 4.21

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.
Draw $\overline{CE} \parallel \overline{DA}$. Now AECD is a parallelogram.
$\therefore$ EC = 5 cm, AE = DC = 4 cm, EB = 3cm.

Step 2 : Draw a line segment AB = 7 cm.

Step 3 : Mark E on $\overline{AB}$ such that AE = 4 cm. [$\because$ DC = 4 cm]

Step 4 : With B and E as centres draw two arcs of radius 5 cm and let them cut at C.

Step 5 : Join $\overline{BC}$ and $\overline{EC}$.

Step 6 : With C and A as centres and with 4 cm and 5 cm as radii draw two arcs. Let them cut at D.

Step 7 : Join $\overline{AD}$ and $\overline{CD}$.

ABCD is the required trapezium.

Step 8 : From D draw $\overline{DF} \perp \overline{AB}$ and measure the length of DF.

DF = h = 4.8 cm. AB = a = 7 cm, CD = b = 4 cm.

Calculation of area:

In the trapezium ABCD, a = 7 cm, b = 4 cm and h = 4.8 cm.

Area of the trapezium ABCD $= \frac{1}{2} h(a+b)$

$= \frac{1}{2}(4.8)\ (7+4)$

$= \frac{1}{2} \times 4.8 \times 11$

$= 2.4 \times 11$

$= 26.4$ cm^2.

4.3.8 Isosceles trapezium

In Fig. 4.22 ABCD is an isosceles trapezium

In an isosceles trapezium,

(i) The non parallel sides are equal in measurement i.e., AD = BC.

(ii) $\angle A = \angle B$.

and $\angle ADC = \angle BCD$

(iii) Diagonals are equal in length i.e., AC = BD

(iv) AE = BF, (DB ⊥ AB, CF ⊥ BA)

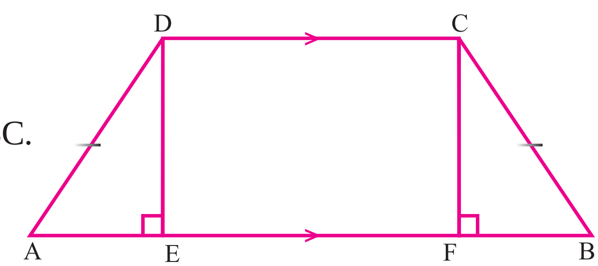


Fig. 4.22

To construct an isosceles trapezium we need only **three independent** measurements as we have two conditions such as

(i) One pair of opposite sides are parallel.

(ii) Non - parallel sides are equal.

4.3.9. Construction of isosceles trapezium

Example 4.10

Construct an isosceles trapezium ABCD in which $\overline{AB}$ is parallel to $\overline{DC}$, AB = 11 cm, DC = 7 cm, AD = BC = 6 cm and calculate its area.

Solution

Given:

$\overline{AB}$ is parallel to $\overline{DC}$, AB = 11 cm,

DC = 7 cm, AD = BC = 6 cm.

Rough Diagram

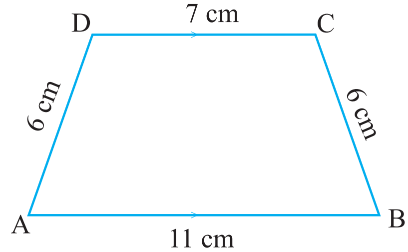


Fig. 4.23

To construct an isosceles trapezium

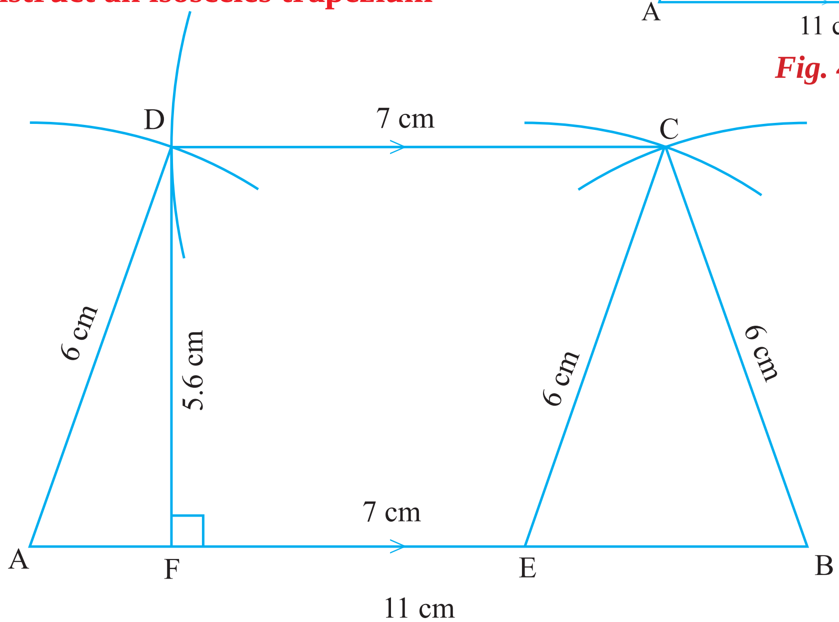


Fig. 4.24

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AB = 11 cm.

Step 3 : Mark E on $\overline{AB}$ such that AE = 7 cm (since DC = 7 cm)

Step 4 : With E and B as centres and (AD = EC = 6 cm) radius 6 cm draw two arcs. Let them cut at C.

Step 5 : Join $\overline{BC}$ and $\overline{EC}$.

Step 6 : With C and A as centres draw two arcs of radii 7 cm and 6 cm respectively and let them cut at D.

Step 7 : Join $\overline{AD}$ and $\overline{CD}$.

ABCD is the required isosceles trapezium.

Step 8 : From D draw $\overline{DF} \perp \overline{AB}$ and measure the length of DF.

DF = h = 5.6 cm. AB = a = 11 cm and CD = b = 7 cm.

Calculation of area:

In the isosceles trapezium ABCD, a = 11 cm, b = 7 cm and h = 5.6 cm.

$$\text{Area of the isosceles trapezium ABCD} = \frac{1}{2}h\,(a+b)$$
$$= \frac{1}{2}(5.6)\,(11+7)$$
$$= \frac{1}{2}\times 5.6\times 18$$
$$= 50.4\ \text{cm}^2.$$

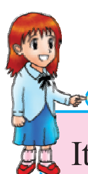
EXERCISE 4.2

I. Construct trapezium PQRS with the following measurements. Find also its area.

1. $\overline{PQ}$ is parallel to $\overline{SR}$, PQ = 6.8 cm, QR = 7.2 cm, PR = 8.4 cm and RS = 8 cm.
2. $\overline{PQ}$ is parallel to $\overline{SR}$, PQ = 8 cm, QR = 5 cm, PR = 6 cm and RS = 4.5 cm.
3. $\overline{PQ}$ is parallel to $\overline{SR}$, PQ = 7 cm, $\angle$ Q = 60°,QR = 5 cm and RS = 4 cm.
4. $\overline{PQ}$ is parallel to $\overline{SR}$, PQ = 6.5 cm, QR = 7 cm, $\angle$ PQR = 85° and PS = 9 cm.
5. $\overline{PQ}$ is parallel to $\overline{SR}$, PQ = 7.5 cm, PS = 6.5 cm, $\angle$ QPS = 100° and $\angle$ PQR = 45°.
6. $\overline{PQ}$ is parallel to $\overline{SR}$, PQ = 6 cm, PS = 5 cm, $\angle$ QPS = 60° and $\angle$ PQR = 100°.
7. $\overline{PQ}$ is parallel to $\overline{SR}$, PQ = 8 cm, QR = 5 cm, RS = 6 cm and SP = 4 cm.
8. $\overline{PQ}$ is parallel to $\overline{SR}$, PQ = 4.5 cm, QR = 2.5 cm, RS =3 cm and SP = 2 cm.

II. Construct isosceles trapezium ABCD with the following measurements and find its area.

1. $\overline{AB}$ is parallel to $\overline{DC}$, AB = 9 cm, DC = 6 cm and AD = BC = 5 cm.
2. $\overline{AB}$ is parallel to $\overline{DC}$, AB = 10 cm, DC = 6 cm and AD = BC = 7 cm.



Do you know?

It is interesting to note that many of the properties of quadrilaterals were known to the ancient Indians. Two of the geometrical theorems which are explicitly mentioned in the **Boudhayana Sutras** are given below:

i) The diagonals of a rectangle bisect each other. They divide the rectangle into four parts, two and two.

ii) The diagonals of a Rhombus bisect each other at right angles.

4.4 Parallelogram

4.4.1. Introduction

In the class VII we have come across parallelogram. It is defined as follows:

A quadrilateral in which the opposite sides are parallel is called a parallelogram.

Consider the parallelogram BASE given in the Fig. 4.25,

Then we know its properties

(i) $\overline{BA} \parallel \overline{ES}$; $\overline{BE} \parallel \overline{AS}$

(ii) BA = ES , BE = AS

(iii) Opposite angles are equal in measure.

$\angle BES = \angle BAS$; $\angle EBA = \angle ESA$

(iv) Diagonals bisect each other.

OB = OS; OE = OA, but BS $\neq$ AE.

(v) Sum of any two adjacent angles is equal to 180°.

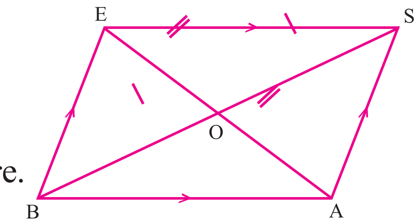


Fig. 4.25

Now, let us learn how to construct a parallelogram, and find its area.

4.4.2 Area of a parallelogram

Let us cut off the red portion (a right angled triangle EFS) from the parallelogram FAME. Let us fix it to the right side of the figure FAME. We can see that the resulting figure is a rectangle. See Fig. 4.27.

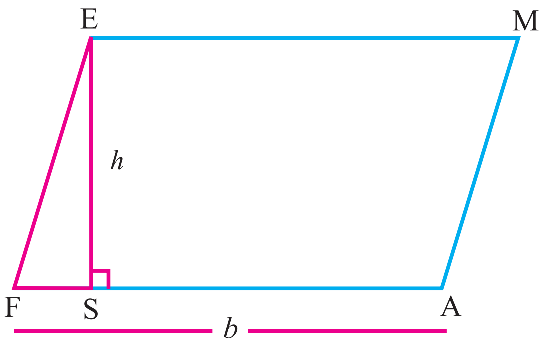


Fig. 4.26

We know that the area of a rectangle having length b units and height h units is given by A = bh sq. units.

Here, we have actually converted the parallelogram FAME into a rectangle. Hence, the area of the parallelogram is A = bh sq. units where 'b' is the base of the parallelogram and 'h' is the perpendicular distance between the parallel sides.

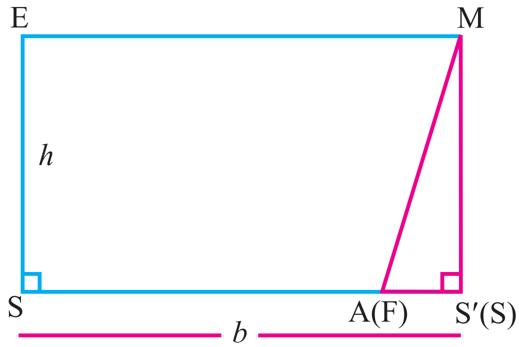


Fig. 4.27

MATHEMATICS

4.4.3 Construction of a parallelogram

Parallelograms are constructed by splitting up the figure into suitable triangles. First a triangle is constructed from the given data and then the fourth vertex is found. We need **three independent** measurements to construct a parallelogram.

We can construct a parallelogram when the following measurements are given .

- **(i) Two adjacent sides, and one angle**
- **(ii) Two adjacent sides and one diagonal**
- **(iii) Two diagonals and one included angle**
- **(iv) One side, one diagonal and one angle.**

4.4.4 Construction of a parallelogram when two adjacent sides and one angle are given

Example 4.11

Construct a parallelogram ABCD with AB = 6 cm, BC = 5.5 cm and $\angle ABC = 80°$ and calculate its area.

Solution

Given: AB = 6 cm, BC = 5.5 cm and $\angle ABC = 80°$.

Rough Diagram

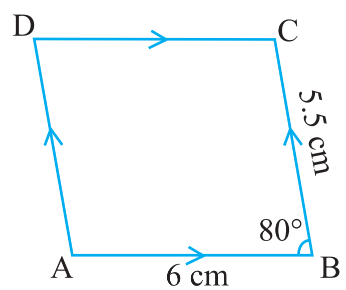


Fig. 4.28

To construct a parallelogram

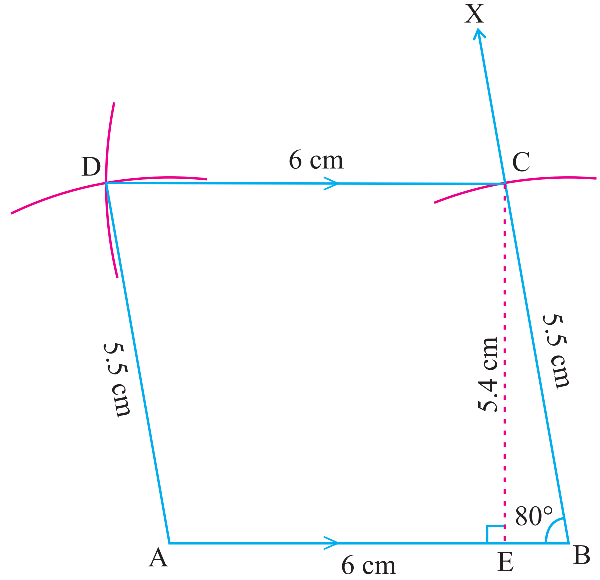


Fig. 4.29

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AB = 6 cm.

Step 3 : At *B* on $\overline{AB}$ make $\angle ABX$ whose measure is 80°.

Step 4 : With B as centre draw an arc of radius 5.5 cm and let it cuts $\overrightarrow{BX}$ at C.

Step 5 : With Cand A as centres draw arcs of radii 6 cm and 5.5 cm repectively and let them cut at D.

Step 6 : Join $\overline{AD}$ and $\overline{CD}$.
ABCD is the required parallelogram.

Step 7 : From C draw $\overline{CE} \perp \overline{AB}$ and measure the length of CE.
CE = h = 5.4 cm. AB = b = 6 cm.

Calculation of area:

In the parallelogram ABCD, b = 6 cm and h = 5.4 cm.

Area of the parallelogram ABCD = $b \times h$ = 6 × 5.4

= 32.4 cm².

4.4.5. Construction of parallelogram when two adjacent sides and one diagonal are given

Example 4.12

Construct a parallelogram ABCD with AB = 8 cm, AD = 7 cm and BD = 9 cm and find its area.

Solution

Given: AB = 8 cm, AD = 7 cm and BD = 9 cm.

To construct a parallelogram

Rough Diagram

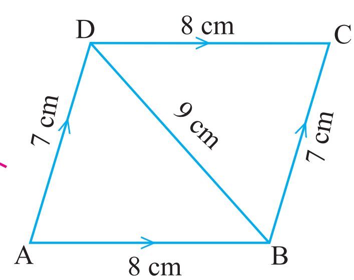


Fig. 4.30

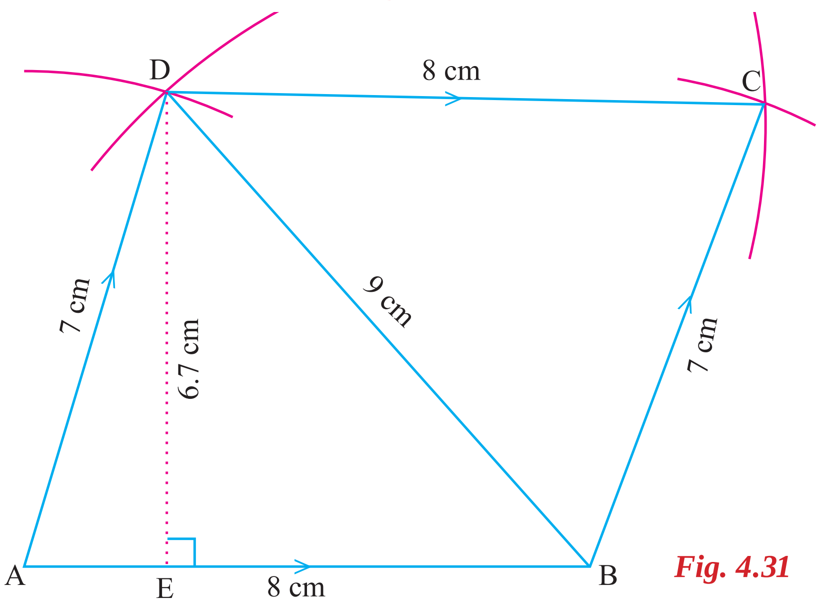


Fig. 4.31

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AB = 8 cm.

Step 3 : With A and B as centres draw arcs of radii 7 cm and 9 cm respectively and let them cut at D.

Step 4 : Join $\overline{AD}$ and $\overline{BD}$.

Step 5 : With B and D as centres draw arcs of radii 7 cm and 8 cm respectively and let them cut at C.

Step 6 : Join $\overline{CD}$ and $\overline{BC}$.

ABCD is the required parallelogram.

Step 7 : From D draw $\overline{DE} \perp \overline{AB}$ and measure the length of DE.

DE = h = 6.7 cm. AB = DC= b = 8 cm

Calculation of area:

In the parallelogram ABCD, b = 8 cm and h = 6.7 cm.

Area of the parallelogram ABCD $= b \times h$

$= 8 \times 6.7 = 53.6 \text{ cm}^2$.

4.4.6. Construction of a parallelogram when two diagonals and one included angle are given

Example 4.13

Draw parallelogram ABCD with AC = 9 cm, BD = 7 cm and $\angle$AOB = 120° where $\overline{AC}$ and $\overline{BD}$ intersect at 'O' and find its area.

Solution

Given: AC = 9 cm, BD = 7 cm and $\angle$ AOB = 120°.

Rough Diagram

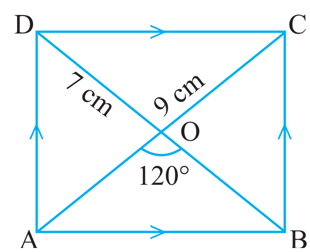


Fig. 4.32

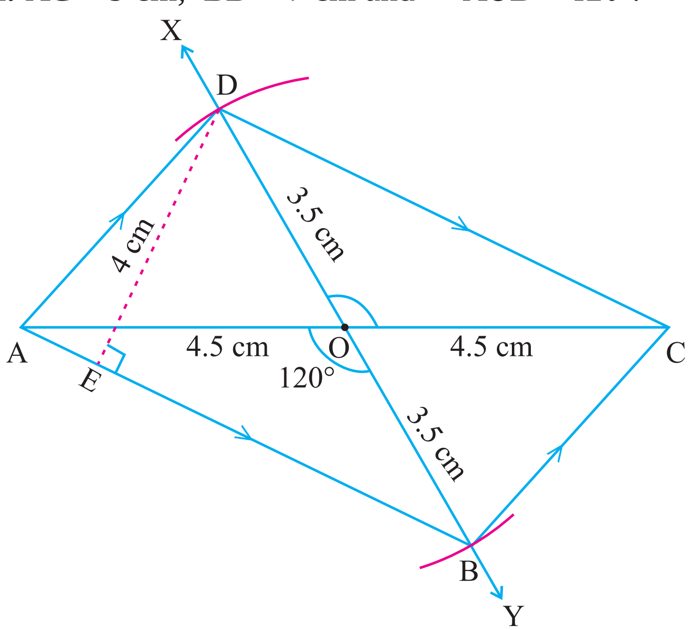


Fig. 4.33

To construct a parallelogram

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AC = 9 cm.

Step 3 : Mark 'O' the midpoint of $\overline{AC}$.

Step 4 : Draw a line $\overleftrightarrow{XY}$ through 'O' which makes $\angle AOY = 120°$.

Step 5 : With O as centre and 3.5 cm as radius draw two arcs on $\overleftrightarrow{XY}$ on either sides of $\overline{AC}$ cutting $\overrightarrow{OX}$ at D and $\overrightarrow{OY}$ at B.

Step 6 : Join $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$.
ABCD is the required parallelogram.

Step 7 : From D draw $\overline{DE} \perp \overline{AB}$ and measure the length of DE.
DE = h = 4 cm. AB = b = 7 cm.

Calculation of area:

In the parallelogram ABCD, b = 7 cm and h = 4 cm.

Area of the parallelogram ABCD = $b \times h = 7 \times 4 = 28$ cm^2.

4.4.7. Construction of a parallelogram when one side, one diagonal and one angle are given

Example 4.14

Construct a parallelogram ABCD, AB = 6 cm, $\angle ABC = 80°$ and AC = 8 cm and find its area.

Rough Diagram

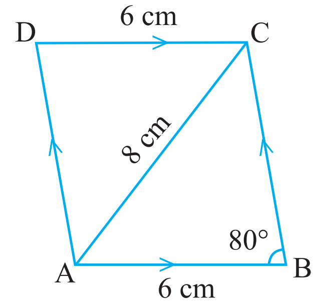


Fig. 4.34

Solution

Given: AB = 6 cm, $\angle$ ABC = 80° and AC = 8 cm.

To construct a parallelogram

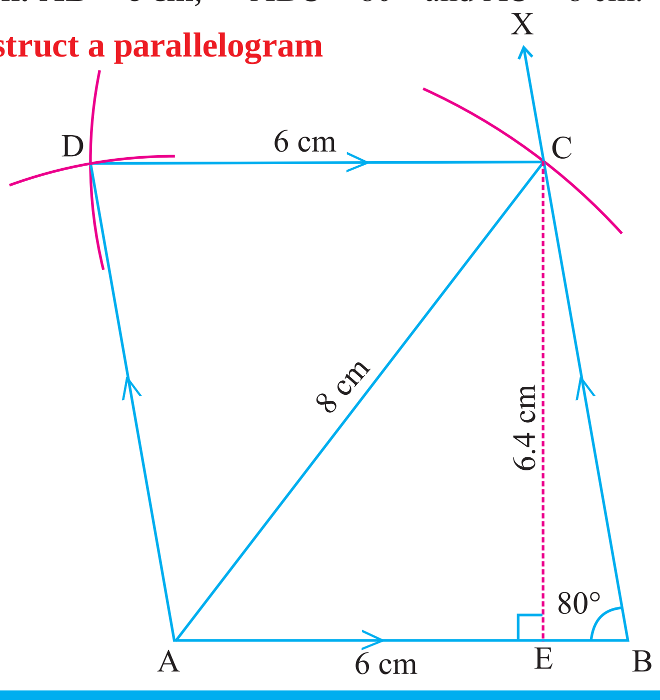


Fig. 4.35

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AB = 6 cm

Step 3 : At B on $\overline{AB}$ make $\angle ABX$ whose measure is 80°.

Step 4 : With A as centre and radius 8 cm draw an arc. Let it cut $\overrightarrow{BX}$ at C.

Step 5 : Join $\overline{AC}$.

Step 6 : With C as centre draw an arc of radius 6 cm.

Step 7 : With A as centre draw another arc with radius equal to the length of BC. Let the two arcs cut at D.

Step 8 : Join $\overline{AD}$ and $\overline{CD}$.

ABCD is the required parallelogram.

Step 9 : From C draw $\overline{CE} \perp \overline{AB}$ and measure the length of CE.

CE = h = 6.4 cm. AB = b = 6 cm.

Calculation of area:

In the parallelogram ABCD, b = 6 cm and h = 6.4 cm.

Area of the parallelogram ABCD $= b \times h$

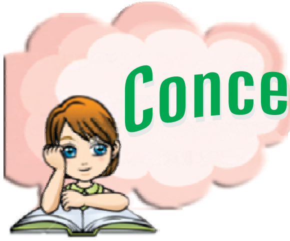
$= 6 \times 6.4$

$= 38.4 \text{ cm}^2$.

EXERCISE 4.3

Draw parallelogram ABCD with the following measurements and calculate its area.

1. AB = 7 cm, BC = 5 cm and $\angle$ ABC = 60°.
2. AB = 8.5 cm, AD = 6.5 cm and $\angle$ DAB = 100°.
3. AB = 6 cm, BD = 8 cm and AD = 5 cm.
4. AB = 5 cm, BC = 4 cm, AC = 7 cm.
5. AC = 10 cm, BD = 8 cm and $\angle$ AOB = 100° where $\overline{AC}$ and $\overline{BD}$ intersect at ‘O’.
6. AC = 8 cm, BD = 6 cm and $\angle$ COD = 90° where $\overline{AC}$ and $\overline{BD}$ intersect at ‘O’.
7. AB = 8 cm, AC = 10 cm and $\angle$ ABC = 100°.
8. AB = 5.5 cm, $\angle$ DAB = 50° and BD = 7 cm.

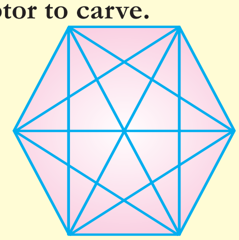


Concept Summary

- A quadrilateral is a plane figure bounded by four line segments.
- To construct a quadrilateral, five independent measurements are necessary.
- A quadrilateral with one pair of opposite sides parallel is called a trapezium.
- To construct a trapezium four independent measurements are necessary.
- If non-parallel sides are equal in a trapezium, it is called an isosceles trapezium.
- To construct an isosceles trapezium three independent measurements are necessary.
- A quadrilateral with each pair of opposite sides parallel is called a parallelogram.
- To construct a parallelogram three independent measurements are necessary.
- 'The area of a quadrilateral, $A = \frac{1}{2} d (h_1 + h_2)$ sq. units, where 'd' is the diagonal, 'h_1' and 'h_2' are the altitudes drawn to the diagonal from its opposite vertices.
- The area of a trapezium, $A = \frac{1}{2} h (a + b)$ sq. units, where 'a' and 'b' are the lengths of the parallel sides and 'h' is the perpendicular distance between the two parallel sides.
- The area of a parallelogram, $A = bh$ sq. units, where 'b' is the base of the parallelogram and 'h' is the perpendicular distance between the parallel sides.

Interesting Information

- **The golden rectangle is a rectangle which has appeared in art and architecture through the years. The ratio of the lengths of the sides of a golden rectangle is approximately 1 : 1.6. This ratio is called the golden ratio. A golden rectangle is pleasing to the eyes. The golden ratio was discovered by the Greeks about the middle of the fifth century B.C.**
- **The Mathematician Gauss, who died in 1855, wanted a 17-sided polygon drawn on his tombstone, but it too closely resembled a circle for the sculptor to carve.**
- **Mystic hexagon: A mystic hexagon is a regular hexagon with all its diagonals drawn.**



ANSWERS

Chapter 1. Number System

Exercise 1.1

1. i) A ii) C iii) B iv) D v) A

2. i) Commutative ii) Associative iii) Commutative

   iv) Additive identity v) Additive inverse

3. i) Commutative ii) Multiplicative identity

   iii) Multiplicative Inverse iv) Associative

   v) Distributive property of multiplication over addition

6. i) $\frac{-505}{252}$ ii) $\frac{-1}{14}$

Exercise 1.2

1. i) $\frac{13}{15}$ ii) $\frac{23}{84}$ iii) $\frac{117}{176}$ iv) $\frac{53}{24}$

2. i) $\frac{31}{70}, \frac{51}{140}$ ii) $\frac{111}{110}, \frac{243}{220}$ iii) $\frac{17}{30}, \frac{9}{20}$ iv) $\frac{-1}{24}, \frac{1}{12}$

3. i) $\frac{3}{8}, \frac{5}{16}, \frac{9}{32}$ ii) $\frac{41}{60}, \frac{83}{120}, \frac{167}{240}$

   iii) $\frac{7}{12}, \frac{1}{8}, \frac{-5}{48}$ iv) $\frac{5}{48}, \frac{11}{96}, \frac{23}{192}$

Note: In fthe above problems 1, 2 and 3; the given answers are one of the possibilities.

Exercise 1.3

1. i) A ii) B iii) C iv) A v) B

2. i) $2\frac{7}{24}$ ii) $\frac{16}{17}$ iii) $\frac{11}{32}$ iv) $1\frac{7}{18}$ v) $\frac{-8}{19}$

   vi) $4\frac{23}{32}$ vii) 4 viii) $-5\frac{41}{60}$

Exercise 1.4

1. i) C ii) B iii) A iv) D v) C

   vi) A vii) B viii) B ix) B x) D

2. i) $\frac{-1}{64}$ ii) $\frac{1}{64}$ iii) 625 iv) $\frac{2}{675}$ v) $\frac{1}{3^{22}}$

   vi) 54 vii) 1 viii) $256\ p^q$ ix) 231 x) $5\frac{1}{3}$

3. i) 5 ii) $\frac{1}{2}$ iii) 29 iv) 1 v) $5\frac{1}{16}$ vi) $\frac{6}{7^{21}}$

4. i) $m = 2$ ii) $m = 3$ iii) $m = 3$ iv) $m = 3$ v) $m = -6$ vi) $m = \frac{1}{4}$

5. a) i) 4 ii) 4 iii) 256 iv) 64 v) $\frac{1}{4}$

5. b) i) 4 ii) 2187 iii) 9 iv) 6561 v) $\frac{1}{9}$

Exercise 1.5

1. (ii), (iii), (v) are not perfect squares.

2. i) 4 ii) 9 iii) 1 iv) 5 v) 4

3. i) 64 ii) 16 iii) 81

4. i) 1 + 3 + 5 + 7 + 9 +11 + 13 ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17

   iii) 1 + 3 + 5 + 7 + 9 iv) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21

5. i) $\frac{9}{64}$ ii) $\frac{49}{100}$ iii) $\frac{1}{25}$ iv) $\frac{4}{9}$ v) $\frac{961}{1600}$

6. i) 9 ii) 49 iii) 0.09 iv) $\frac{4}{9}$ v) $\frac{9}{16}$ vi) 0.36

7. a) $4^2 + 5^2 + \underline{20^2} = 21^2$ b) 10000200001

   $5^2 + \underline{6^2} + 30^2 = 31^2$ 100000020000001

   $6^2 + 7^2 + \underline{42^2} = \underline{43^2}$

Exercise 1.6

1. i) 12 ii) 10 iii) 27 iv) 385

2. i) $\frac{3}{8}$ ii) $\frac{1}{4}$ iii) 7 iv) 4

3. i) 48 ii) 67 iii) 59 iv) 23 v) 57

   vi) 37 vii) 76 viii) 89 ix) 24 x) 56

4. i) 27 ii) 20 iii) 42 iv) 64 v) 88

   vi) 98 vi) 77 viii) 96 ix) 23 x) 90

5. i) 1.6 ii) 2.7 iii) 7.2 iv) 6.5 v) 5.6

   vi) 0.54 vii) 3.4 viii) 0.043

6. i) 2 ii) 53 iii) 1 iv) 41 v) 31

7. i) 4 ii) 14 iii) 4 iv) 24 v) 149

8. i) 1.41 ii) 2.24 iii) 0.13 iv) 0.94 v) 1.04

9. 21 m 10. i) $\frac{15}{56}$ ii) $\frac{46}{59}$ iii) $\frac{23}{42}$ iv) $1\frac{13}{76}$

Exercise 1.7

1. i) A ii) D iii) B iv) A v) B
   vi) D vii) A viii) A ix) A x) D
2. ii) 216 iii) 729 v) 1000
3. i) 128 ii) 100 v) 72 vi) 625
4. i) 3 ii) 2 iii) 5 iv) 3 v) 11 vi) 5
5. i) 3 ii) 2 iii) 3 iv) 5 v) 10
6. i) 9 ii) 7 iii) 8 iv) 0.4 v) 0.6
   vi) 1.75 vii) – 1.1 viii) – 30
7. 2.7 cm

Exercise 1.8

1. i) 12.57 ii) 25.42 kg iii) 39.93 m
   iv) 56.60 m v) 41.06 m vi) 729.94 km
2. i) 0.052 m ii) 3.533 km iii) 58.294 *l*
   iv) 0.133 gm v) 365.301 vi) 100.123
3. i) 250 ii) 150 iii) 6800 iv) 10,000
   v) 36 lakhs vi) 104 crores
4. i) 22 ii) 777 iii) 402 iv) 306 v) 300 vi) 10,000

Exercise 1.9

1. i) 25, 20, 15 ii) 6, 8, 10 iii) 63, 56, 49
   iv) 7.7, 8.8, 9.9 v) 15, 21, 28 vi) 34, 55, 89
   vii) 125, 216, 343
2. a) 11 jumps b) 5 jumps
3. a) 10 rows of apples = 55 apples b) 210 apples

Rows	1	2	3	4	5	6	7	8	9
Total apples	1	3	6	10	15	21	28	36	45

Chapter 2. Measurements

Exercise 2.1

1. i) C ii) B iii) A iv) D v) A
   vi) D vii) B viii) C ix) A x) C
2. i) 180 cm, 1925 cm^2 ii) 54 cm, 173.25 cm^2
   iii) 32.4 m, 62.37 m^2 iv)25.2 m, 37.73 m^2
3. i) 7.2 cm, 3.08 cm^2 ii) 144 cm, 1232 cm^2
   iii) 216 cm, 2772 cm^2 iv) 288m, 4928 m^2
4. i)350 cm, 7546 cm^2 ii) 250 cm, 3850 cm^2
   iii)150 m, 1386 m^2 iv) 100 m, 616 m^2
5. 77 cm^2, 38.5 cm^2 6. Rs.540

Exercise 2.2

1. i) 32 cm ii) 40 cm iii) 32.6 cm iv) 40 cm v) 98 cm
2. i) 124 cm^2 ii) 25 m^2 iii) 273 cm^2 iv) 49.14 cm^2 v) 10.40m^2
3. i) 24 m^2 ii) 284 cm^2 iii) 308 cm^2
   iv) 10.5 cm^2 v) 135.625 cm^2 vi) 6.125cm^2
4. 770 cm^2 5. 1286 m^2 6. 9384 m^2 7. 9.71 cm^2
8. 203 cm^2 9. 378 cm^2 10. i) 15,100 m^2, ii) 550000 m^2

Chapter 3. Geometry

Revision Exercise

1. $y° = 52°$ 2. $x° = 40°$ 3. $\angle A = 110°$ 4. $x° = 40°$
5. $x° = 105°$ 6.i) Corresponding angle, ii) Alternate angle, iii) Coresponding angle

Exercise 3.1

1. i) B ii) A iii) A iv) B v) A
2. $x° = 65°$ 3. $x° = 42°$
5. i) $x° = 58°$, $y° = 108°$ ii) $x° = 30°, y° = 30°$ iii) $x° = 42°, y° = 40°$
6. $x° = 153°$, $y° = 132°$, $z° = 53°$.

Exercise 3.2

1.i)C ii) C iii) C iv) C v) B vi) A vii) B
2. $x° = 66°$, $y° = 132°$ 3. $x° = 70°$
4. $x° = 15°$ 7. $x° = 30°$, $y° = 60°$, $z° = 60°$

Play with Numbers

Sequential Inputs of numbers with 8

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$
$$1234 \times 8 + 4 = 9876$$
$$12345 \times 8 + 5 = 98765$$
$$123456 \times 8 + 6 = 987654$$
$$1234567 \times 8 + 7 = 9876543$$
$$12345678 \times 8 + 8 = 98765432$$
$$123456789 \times 8 + 9 = 987654321$$

Sequential 8's with 9

$$9 \times 9 + 7 = 88$$
$$98 \times 9 + 6 = 888$$
$$987 \times 9 + 5 = 8888$$
$$9876 \times 9 + 4 = 88888$$
$$98765 \times 9 + 3 = 888888$$
$$987654 \times 9 + 2 = 8888888$$
$$9876543 \times 9 + 1 = 88888888$$
$$98765432 \times 9 + 0 = 888888888$$

Without 8

$$12345679 \times 9 = 111111111$$
$$12345679 \times 18 = 222222222$$
$$12345679 \times 27 = 333333333$$
$$12345679 \times 36 = 444444444$$
$$12345679 \times 45 = 555555555$$
$$12345679 \times 54 = 666666666$$
$$12345679 \times 63 = 777777777$$
$$12345679 \times 72 = 888888888$$
$$12345679 \times 81 = 999999999$$

Numeric Palindrome with 1's

$$1 \times 1 = 1$$
$$11 \times 11 = 121$$
$$111 \times 111 = 12321$$
$$1111 \times 1111 = 1234321$$
$$11111 \times 11111 = 123454321$$
$$111111 \times 111111 = 12345654321$$
$$1111111 \times 1111111 = 1234567654321$$
$$11111111 \times 11111111 = 123456787654321$$
$$111111111 \times 111111111 = 12345678987654321$$